Financial Frictions, Technology Diffusion and Growth

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Abstract

Using an endogenous growth model as a framework, I introduce capital and a banking sector to the economy and analyze the effects of financial frictions. In a context with asymmetric information between banks and firms, I assume that it is more costly for banks to monitor projects (firms) that are closer to the frontier of knowledge. I find that this friction has considerable effects on economic growth and welfare.

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1 Introduction

A large literature on economic development documents a positive relationship between financial depth and economic growth. On the empirical side, King and Levine (1993), Beck, Levine and Loayza (2000) and Christopoulos and Tsionas (2004) find that financial development (measured by financial depth indicators and size of the financial and banking systems) increases per capita growth rates. Further, Benhabib and Spiegel (2000) find that there is a positive correlation between factor productivity growth and financial development.

Figure 1 shows this fact using data for the period (1960-2010). It depicts the relationship between banks' interest margin (the difference between lending and deposits rates) and long run growth rate (average GDP per capita growth rates). As shown, countries less financially developed (those with higher spreads) tend to have lower growth rates.

This paper tries to give a theoretical explanation to this relationship: it analyzes the effects of financial frictions on economic growth. Using Perla and Tonetti (2014) growth model as a baseline, it introduces asymmetric information in the banking sector and assumes that projects that are close to the frontier are harder to monitor by the bankers. As a consequence, banks have to spend more resources monitoring projects to be sure that firms pay what they must pay. I find that this friction has considerable effects: depending on how high the cost is, the long run growth rate and welfare can drop dramatically.

This paper is related to a long theoretical literature on financial development and growth.[1] Particularly, there is a recent series of papers addressing the relevance of financial frictions. Surprisingly, most of these papers focus on the stationary level of productivity rather than the productivity growth rate: Jeong and Townsend (2007), Quintin (2008), Amaral and Quintin (2010), Buera and Shin (2010), Greenwood, Sanchez and Wang (2010), Midrigan and Xu (2010), Buera, Kaboski and Shin (2011) and Moll (2012). As this literature focuses on stationary equilibria, it does not address the effects of financial frictions on innovation decisions and growth.[2]

Moreover, the mentioned literature does not take into account that the degree of financial frictions might be different across projects. Information asymmetry between investors and firms tends to be more important for projects in the frontier of knowledge. According to the existing evidence, these kind of projects, which are generally financed through venture capital, are monitored with much higher intensity than those that are not in the frontier.[3]

This paper differs from the existing literature in two main aspects: (i) it addresses the effects of financial frictions on the innovation decisions of firms and, hence, on long run growth; and (ii) it introduces the assumption that banks have to pay a higher cost to monitor more sophisticated projects.

The rest of the paper goes on as follows. Section 2 presents the model. Section 3 defines

[1]Among the earliest contributions we can mention Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Benarjee and Newman (1993), Galor and Zeira (1993), Raymond and Jovanovic (1993), and Aghion and Bolton (1997).

[2]Chiu, Meh and Wright (2011) develop a growth model but studies frictions in the ideas/patents markets.

[3]Gorman and Sahlman (1989)finds that venture capitalists monitor their portfolio companies 19 times a year, whereas Blackwell and Winters (1997) find that banks monitor projects 1 or 2 times per year. Hellmann and Puri (2002) find evidence indicating a higher involvement of venture capitalists in the companies' management relative to other type of lenders.

the equilibrium and shows the balance growth path. In section 4 I solve the model, calibrate it and compare with cross country data. Section 5 shows and solves the constrained planner problem. Section 6 concludes.

2 The model

Time is discrete with an infinite horizon. There are three types of agents: households, banks and firms.

2.1 Households

I represent the households with a representative consumer that faces a very basic problem. He decides whether to save or consume. The only financial instruments to save that are available in the economy are bank deposits D. We assume the following period utility function and a discount factor $\beta \in (0, 1)$.

$$U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}, \sigma > 0 \tag{1}$$

Hence, the household problem is the following:

$$\begin{aligned} \underset{\{C_t, D_{t+1}\}_{t=0}^{\infty}}{max} & \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t) \right\} \\ \text{st. } C_t + D_{t+1} - D_t &= r_t D_t + \Pi_t \\ D_0 &> 0 \end{aligned} \tag{2}$$

Where Π_t represents the aggregate profits of the firms, D_t is the real amount of deposits, R_t is the gross deposit interest rate, and C_t is consumption. From this problem we can get the following first order condition:

$$\left(\frac{C_{t+1}}{C_t} \right)^{\sigma} = (1 + r_t) \beta \tag{3}$$

2.2 Firms

I assume that a firm with productivity z has access to this technology,

$$y_t = \begin{cases} zk_t^{\alpha}, \text{ with prob. } p \\ 0, \text{ with prob. } 1-p \end{cases} \tag{4}$$

Where y_t is the output level, k_t represents the capital invested and $\alpha \in (0,1)$ is the capital-output elasticity. Every period a project can be either successful and generate a positive profit with probability p, or it can fail with probability $1-p$ and produce nothing.

Firms dynamic behavior is similar to the one in Perla and Tonetti (2014). I am assuming the same timing. At time 0 all firms draw a potential productivity level z from the distribution $F_0(z)$. For periods $t = 1, 2, ..,$ at the beginning of each period every firm decides whether to carry out its project with the technology described in (4) or to search for a new project with a different productivity level. Firm's timeline is described in figure 2.

If a firm decides to run its project at time t, it borrows capital from a bank by signing a contract that promises an expected profit equal to $\pi_t(z)$. The project succeeds (fails) with probability p $(1-p)$, the firm reveal its type to the bank and payments are made. After making the payments the firm goes to the following period with the same level of productivity.

On the other hand, if it decides to search, the firm does not produce at t but draws a new productivity level from the distribution of productivity levels of firms that decided to produce at time t. Note that I assume that the firms that decide to search can not produce at time t, but they can do it with the new technology at $t+1$.

2.2.1 Recursive problem

The value function to be solved by each firm is the following:

$$V_t(z) = max\left\{ \pi_t(z) + \frac{1}{1+r_t} V_{t+1}(z); \frac{1}{1+r_t} \int V_{t+1}(z')\, dF_{t+1}(z') \right\} \tag{5}$$

$$\text{st. } F_{t+1} = \Gamma(F_t)$$

Where r is the net interest rate, Γ is the law of motion of the distribution of probabilities. As we can see, this problem in the way it is presented in (5) is very complicated as one state variable is the distribution of levels of productivity. Following Perla and Tonetti (2014), since the only cost of innovating is the opportunity cost of not producing, firms with lower productivity levels will be the ones innovating. In fact, there is a cutoff productivity level below which firms decide to innovate. This in turn implies that in equilibrium the distribution of productivity levels will be shifting to the right by a sequence of truncations at those cutoff levels.[4]

Moreover, if I assume that the initial productivity distribution F_0 is Pareto with tail parameter κ this problem becomes much simpler. The value function in this case becomes,[5]

[4] I am skipping the details here, please refer to Perla and Tonetti (2014).

[5] This is related to the fact that a truncation of a Pareto distribution with lowest productivity level m_t and tail parameter κ at the level m_{t+1} is Pareto with parameters (m_{t+1}, κ).

$$V(z,m_t) = max\left\{\pi(z,m_t) + \frac{1}{1+r(m_t,m_{t+1})}V(z,m_{t+1}),\right.$$
$$\left.\frac{1}{1+r(m_t,m_{t+1})}\int V(z',m_{t+1})\,dF_{m_{t+1}}(z')\right\} \quad (6)$$
$$m_{t+1} = f(m_t)$$

Where m_t represents the lowest productivity level in the support of F_t. Hence, with a Pareto distribution I only need to keep track of one parameter (m_t) instead of the entire distribution.

2.3 Banks

There is a continuum of banks and the banking sector is competitive. There are no entry costs. Firms must go to the banking sector to get funds to invest in their projects. I will assume that the outcome of the project (whether it succeeds or fails) is firm's private information. Therefore, given the assumption of limited liability, if a project turns out to be successful, the firm has incentives to misreport the outcome of the project: by reporting a failure the firm will pay a lower amount to the bank.

In order to avoid this behavior, the bank can decide to monitor the firm to check whether the firm is reporting the real outcome of the project. In order to monitor the firm, I assume that the bank needs to pay a monitoring cost. I will assume that the information asymmetry is worse when the project is closer to the frontier of knowledge and only being carried out by a small fraction of firms. In particular, I will assume that the cost of monitoring per unit of capital lent is given by a function $M(z,m)$ that is homogeneous of degree zero in z and m [6].

Banks are risk neutral and, assuming truth-telling, they have the following profit function for every project they finance with productivity z:

$$\xi = px_{HH} + (1-p)x_{LL} - (r+\delta)k - pM(z,m)ke_H - (1-p)M(z,m)ke_L \quad (7)$$

Where, x_{HH} is the amount the firm must pay if its project succeeds and the firm tells the truth to the bank, x_{LL} is the amount the firm must pay in case the project fails and the firm truthfully reports that outcome, $e_H \in \{0,1\}$ represents the decision of monitoring the project when the firm reveals that the project succeeded, and $e_L \in \{0,1\}$ is the monitoring decision when the firm reveals that the project failed. Finally, k is the amount the bank decides to lend to the firm.

[6] This assumption is important to have an homogeneous problem and apply the usual dynamic programming theorems.

2.3.1 Optimal Contract

Assume that the lowest productivity level of the economy is m, and that a firm with productivity z wants to borrow money from a bank. The optimal debt contract that will emerge has to solve the following problem,

$$\xi(z,m,\pi) \equiv \max_{\{x,e_H,e_L,k\}} \{px_{HH} + (1-p)x_{LL} - (r+\delta)k - pM(z,m)ke_H - (1-p)M(z,m)ke_L\}$$

$$\text{st.}$$

$$0 \le x_{HH} \le zk^{\alpha}$$

$$0 \le x_{LL} \le 0$$

$$0 \le x_{HL} \le zk^{\alpha}$$

$$0 \le x_{LH} \le 0$$

$$(1-e_H)(0-x_{HH}) + e_H(0-x_{LH}) \le 0 - x_{LL}$$

$$(1-e_L)(zk^{\alpha}-x_{LL}) + e_L(zk^{\alpha}-x_{HL}) \le zk^{\alpha} - x_{HH}$$

$$p(zk^{\alpha}-x_{HH}) + (1-p)(0-x_{LL}) = \pi$$

Where x_{HL} is the amount a firm must pay if it is found that its project got a positive payoff but the firm reported that the return was zero. On the other hand, x_{LH} is the amount to be paid when a firm reports a high return when it actually had a null return. π is firm's expected profit and it will be determined in equilibrium. $x_{HH}, x_{LL}, e_H, e_L, k$ were described above. I will call the resulting contract $\Omega(z,m) = \{x_{HH}(z,m), x_{LL}(z,m), x_{HL}(z,m), x_{LH}(z,m), e_H(z,m), e_L(z,m), k(z,m)\}$.

2.3.2 Firm's profit from contract

Since I am assuming that there is no cost of entry in the banking sector and that it is competitive, every bank must make zero profits. Hence, we must have that,

$$\xi(z,m,\pi) = 0 \tag{8}$$

Therefore, firm's profit is defined as,

$$\pi(z,m) = \max_{\pi} \left\{\pi \in \mathbb{R}^{+} : \xi(z,m,\pi) = 0\right\} \tag{9}$$

3 Equilibrium and BGP

Here in this paper I am interested in the effects of financial frictions in the long run growth. Hence, I am going to focus in a balanced growth path. In order to define a balanced growth path (BGP) I am going to look for an equilibrium in which the productivity distribution shifts to right at a constant rate and all its quantiles grow at the same rate. As I mentioned above I will assume that the initial distribution of productivity levels is Pareto with parameter m_0 and κ.

$$F_0(z) = 1 - \left(\frac{m_0}{z}\right)^{\kappa}, \forall z \geq m_0$$

And in a balanced growth path this distribution shifts to the right every period at a rate g. So,

$$F_t(z) = 1 - \left(\frac{g^t m_0}{z}\right)^{\kappa}, \forall z \geq g^t m_0 \tag{10}$$

With this in mind, let's define the resulting competitive equilibrium and the BGP.

Definition 1 *A Recursive competitive equilibrium is an initial distribution of productivity levels F_0, an initial amount of deposits D_0 and a set of policy functions $\{C_t, D_{t+1}\}$, debt contracts with the corresponding profit function $\pi(z, m_t)$, a reservation productivity level $z^* = m_{t+1}$, prices r_t and value function $V(z, m_t)$ such that:*

- *The profit function is derived from the optimal contract;*
- *Given r_t and m_t, $V(z, m_t)$ and z^* solve the firm's problem.*
- *Given r_t, $\{C_t, D_{t+1}\}$ solve the Household problem.*
- *Market clearing:*

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t + \int_{m_{t+1}}^{\infty} (1-p)M(z, m_t)k(z, m_t)dF_{m_{t+1}}(z)$$

Where Y_t, K_t denotes aggregate output and capital, respectively.

Now, let's go to the definition of BGP:

Definition 2 *A BGP competitive equilibrium is a competitive equilibrium in which:*

- *The reservation productivity (and truncation point) m_{t+1} grows at a rate $g > 1$. So $m_{t+1} = gm_t$ for all t.*
- *Y_t, C_t and K_t grow at a rate $f(g) > 1$.*
- *All F_{m_t} quantiles grow at the same rate g.*

4 Solution

In order to describe the solution let's first solve the optimal contract between the bank and a firm with productivity z in an economy with a lower productivity bound of m. The next proposition characterizes the solution to the contract problem.

Proposition 1 *The solution to contract problem between a bank and a firm with productivity z when the productivity distribution has a lower bound m is given by:*

(i) A schedule of payments:

$$x_{HH} = zk^{\alpha} - \frac{\pi}{p} \tag{11}$$

$$x_{HL} = zk^{\alpha} \tag{12}$$

$$x_{LL} = 0 \tag{13}$$

$$x_{LH} = 0 \tag{14}$$

(ii) The amount lent:

$$k = \left[\frac{\alpha p z}{[r + \delta + (1-p)M(z,m)]} \right]^{\frac{1}{1-\alpha}} \tag{15}$$

(iii) Monitoring decision rules:

$$e_L = 1 \tag{16}$$

$$e_H = 0 \tag{17}$$

(iv) A resulting firm's profit given by:

$$\pi(z,m) = (1-\alpha)\, pzk^{\alpha} \tag{18}$$

$$= \phi \left[\frac{z^{\frac{1}{\alpha}}}{[r + \delta + (1-p)M(z,m)]} \right]^{\frac{\alpha}{1-\alpha}} \tag{19}$$

Where $\phi = (1-\alpha)p\,[\alpha p]^{\frac{\alpha}{1-\alpha}}$.

Proof. The solution simply comes from a typical state verification problem. See Appendix. ■

Now that I have the profit function for the firm, I can plug it in problem (6) and find the BGP equilibrium. First, note that in a BGP in which m grows at a rate g, the value function (6) becomes:

$$V(z,m) = max\left\{\pi(z,m) + \frac{1}{1+r}V(z,gm), \frac{1}{1+r}\int\limits_{gm}^{\infty} V(z',gm)\, dF_{gm}(z')\right\} \tag{20}$$

Note that the profit function (19) is homogeneous of degree $1/(1-\alpha)$ so by a classic dynamic programming argument we can show that the value function is homogeneous of degree $1/(1-\alpha)$ as well. With that in mind, and defining $\tilde{z} = \frac{z}{m}$, $\tilde{V}(\tilde{z}) \equiv V(\tilde{z}, 1)$ and $\tilde{\pi}(\tilde{z}) \equiv \pi(\tilde{z}, 1)$ we can simplify the value function (20) in this way:

$$\tilde{V}(\tilde{z}) = max\left\{\tilde{\pi}(\tilde{z}) + \frac{g^{\frac{1}{1-\alpha}}}{1+r}\tilde{V}\left(\frac{\tilde{z}}{g}\right), \frac{g^{\frac{1}{1-\alpha}}}{1+r}\int\limits_{g}^{\infty} \tilde{V}\left(\frac{\tilde{z}'}{g}\right)\, dF_g(\tilde{z}')\right\} \tag{21}$$

In the transition from equation (20) to (21) we apply the fact that if z follows a Pareto distribution with parameter κ and lower bound m (or gm), then $\tilde{z}$ distributes Pareto with the same parameter κ and lower bound 1 (or g).

Note that by looking at the second term of the max operator of (21), it is easy to see that the normalized value of search is a constant. So a natural guess is that in a BGP the value function for firms with normalized productivity levels $\tilde{z} \in [1, g]$ is a constant W. In the next proposition we apply this guess and solve the model:

Proposition 2 *Given an initial distribution F_0, that is Pareto with parameters κ and $m_0 > 0$, an equilibrium exists*[7] *with the following characteristics:*

- *The growth rate of m (g) and the constant of the value of search (W) are the solution to the following system:*

$$\tilde{\pi}(g) = \beta g^{\frac{1-\sigma}{1-\alpha}} \int\limits_{g}^{\infty} \tilde{\pi}(\omega)\, \kappa\omega^{-\kappa-1} d\omega \tag{22}$$

$$W = \frac{\tilde{\pi}(g)}{1 - \frac{1}{1+r} g^{\frac{1}{1-\alpha}}} \tag{23}$$

- *C, K, Y grow at a rate $g^{\frac{1}{1-\alpha}}$;*

- *The deposit interest rate is given by: $1 + r = \frac{g^{\frac{\sigma}{1-\alpha}}}{\beta}$;*

- *The average spread in the economy is defined by $\overline{r_k} - (r+\delta)$, where: $\overline{r_k} = \int\limits_{g}^{\infty} \frac{x_{HH}(\tilde{z})}{\tilde{k}(\tilde{z})} dF_g(\tilde{z})$*

[7] For this equilibrium to exists we need $\sigma \geq 1$, $\frac{\delta\tilde{\pi}}{\delta\tilde{z}} \geq 0$, and $\tilde{\pi}(1) < \beta \int\limits_{1}^{\infty} \tilde{\pi}(\omega)\, \kappa\omega^{-\kappa-1} d\omega$.

Proof. See Appendix. ■

In general, equations (23) and (22) do not have a closed form solution. However, there is one particular case for which I can find an explicit equation for the growth rate. This particular case is shown in the next proposition.

Proposition 3 *If the cost of monitoring* $M(z,m)$ *satisfies,*

$$(1-p)M(z,m) = \left[\left(\frac{z}{m}\right)^{\theta} - 1\right](r+\delta)$$

the growth rate of m *is given by:*

$$g = \left[\frac{\beta\kappa}{\kappa - \frac{1-\kappa\theta\alpha}{1-\alpha}}\right]^{\frac{1-\alpha}{(1-\alpha)\kappa+\sigma-1}}$$

Proof. See Appendix. ■

The last proposition shows that there is a negative effect of the financial frictions parameter θ on the long run growth rate: the more costly is to monitor firms in the tail of the distribution, the lower the growth rate. In the next section I calibrate the model to fit the US economy. Further, a cross country comparison is carried out to assess the relevance of this type of frictions.

4.1 Numerical Example: the US economy

With the BGP in hand I can assign values to the parameters and assess the effect of financial frictions on the long run growth of GDP, in our case $g^{1/(1-\alpha)}$. Table 1 shows the values I assigned to some parameters using data from other sources

Table 1: Calibration using other sources

Description	Parameter	Value	Source
Production Function	α	0.35	Standard
Discount Factor	β	0.95	Standard
Depreciation Rate	δ	0.10	Standard
Risk Aversion	σ	3	Standard
Probability of Failure	$1-p$	0.03	Non-performing loans/Total loans

Table 2 shows the calibration using US data. I am calibrating the monitoring cost parameter θ and the distribution tail parameter κ to match the interest margin (or spread) in the banking sector, and the long run growth rate.

Table 2: Calibration using US data

Description	Parameter	Value
Cost Function	θ	0.701
Pareto Distribution	κ	6.648
Target	Data	Model
Growth Rate	2.25%	2.25%
Interest Margin	3.7%	3.7%

With this parameters of the model I can perform following counterfactual exercise. Suppose that all deep parameters are constant and modify θ to get an estimate of what the growth rate would be if we changed the ability of the banking sector to deal with the asymmetric information problem.

Figure 3 shows the growth rate of GDP, the welfare loss (expressed in % of consumption the household is willing to reduce to avoid having monitoring costs) and the percentage of firms that decide to catch up as a function of the parameter θ. Remember that the higher θ is, the harder is for the bank to monitor firms close to the frontier of knowledge. The figure shows that there is a strong impact on the rate of growth and welfare. In particular, if θ is reduced to zero, the growth rate increases to levels around 3.5%. On the other hand, the household is willing to give up more than 10% of its consumption to perform this change of monitoring technology.

The intuition behind this result is that the presence of this kind of friction makes less attractive for firms to look for new production technologies because, more complicated technologies are more expensive to implement given that the banks have to make more effort to monitor the production process. Hence, firms wait longer to search for new technologies and that makes the growth rate to decline. We can see this fact in the last panel of figure 3.

It is common in the empirical literature to assess the financial depth of a country using the interest margin in the banking sector (average lending rate minus deposit rates). A higher banking interest margin is associated with underdeveloped financial systems. This model provides predictions on the relationship of this indicator with the long run growth rate. Therefore, I can plot the curve in the top panel of figure 3 in the growth-spread space. Figure 4 shows the predictions from the model (the solid line), a scatterplot showing country level data from the Beck et al (1999) and World Development Indicators[8], and fitted values from linear regressions.

4.2 Numerical Example: Cross country calibration

In figure 4 we are assuming that the only source of heterogeneity comes from the differences in θ across countries. Here I relax that assumption and assume that θ and κ can vary. In particular, I perform the same calibration I made for the US to the rest of the countries for which I have data. The histogram of estimates for κ and θ that this exercise yields is shown in figure 5.

One way to assess how well the model identifies the different degrees of financial frictions

[8]World Development Indicators, World Bank

across countries is to compare the θ estimates with other financial depth or efficiency indicators that are generally used in the empirical literature. We do this comparison using the following indicators from Beck etal (2000): Private Credit to GDP ratio, Bank Deposits to GDP ratio, and Bank overhead costs to assets ratio. Figure 6 shows scatterplots comparing these indicators with θ. As we can see, a higher θ is associated with lower financial depth (lower private credit and deposits to GDP ratios), and lower financial efficiency (higher overhead to assets ratio).

By doing this calibration we are basically identifying the heterogeneity of growth rates across countries with only two parameters κ and θ. The parameter κ is related to the search friction in the model: a higher value means a thinner tail and, therefore, a lower probability of finding a high productivity project when innovating. On the other hand, as implied by figure 6, θ is related to financial frictions.

One relevant question that arises is how important financial frictions are according to the model. In order to give an answer to that question we can do a simple exercise. From the calibration we get the model's growth rates as a function of κ_i and θ_i for each country i. Denoting $growth_i$ the country i's growth rate we have that,

$$growth_i = f(\kappa_i, \theta_i)$$

In order to focus on the effects of financial frictions, I fix the parameter κ to the cross country average $\bar{\kappa}$ and compute the growth rate for each country using each country's θ estimate. I denote this growth rate with $growth_i^{\bar{\kappa}}$. Therefore for each i I define,

$$growth_i^{\bar{\kappa}} = f(\bar{\kappa}, \theta_i)$$

A measure of how relevant financial frictions are is the ratio between the variances of $growth_i^{\bar{\kappa}}$ and $growth_i$. Table 3 shows the mentioned ratio,

Table 3: Variance Decomposition

Growth rates	SD	Variance Share
$growth_i$	1.44%	
$growth_i^{\bar{\kappa}}$	0.81%	31%

Hence, according to this exercise 31% of the cross country growth variance is explained by financial frictions. Nevertheless, it should be noted that, as we are abstracting from several sources of heterogeneity, this value constitutes an upper bound estimate to the mentioned ratio.

5 Welfare

In this section we characterize the first best growth rate of the described economy. As in any model of technology diffusion there is a clear source that makes the equilibrium to deviate from the socially optimal. There is inefficiency because the firms do not internalize the positive externality that innovating generates. By shifting to the right the productivity distribution, firms make new technologies available to new imitators.

In order to get the optimal growth rate we will focus on a balanced growth path with the same characteristics as the equilibrium. Hence, I will assume that the planner faces the same asymmetric information problem that banks face in equilibrium.

The planner has to choose the representative agent consumption, the rate of capital accumulation, the allocation of capital among firms, and the innovation activities by firms. The allocation of capital follows the same timing as in equilibrium. First, the planner assigns capital to each firm based on their productivity levels z. After that, a shock is realized to each one of the firms: with probability p the firm produces and with probability $1-p$ the project fails. The planner does not know the outcome of the project of each firm but asks each one them to report it. To check whether the firms are telling the truth the planner can monitor them using the same technology that banks have in the equilibrium.

Taking into account what I mentioned above, and after some simplifications, the planner's problem can be expressed as,

$$
\begin{aligned}
W(K_t, m_t) &= \max_{\{K_{t+1}, g\}} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \beta W(K_{t+1}, m_{t+1}) \right\} \\
&\text{st.} \\
m_{t+1} &= g m_t \\
C_t &= Y_t - K_{t+1} + (1-\delta) K_t \\
Y_t &= \max_{\{k_t(z)\}} \left\{ \int_{gm_t}^{\infty} \left[p z k_t(z)^{\alpha} - (1-p) M(z, m_t) k_t(z) \right] dF_{gm_t}(z) \right\} \\
&\quad \text{st.} \int_{gm_t}^{\infty} k_t(z) dF_{gm_t}(z) = K_t \\
g &> 1
\end{aligned}
$$

The next proposition characterizes the optimal growth rate.

Proposition 4 *The first best steady state growth rate of m that solves the planner problem is given by,*

$$
g = \left[\beta \frac{\frac{1}{1-\alpha} + \epsilon}{\epsilon} \right]^{\frac{1-\alpha}{\sigma - 1}} \tag{24}
$$

where,

$$
\epsilon = -\frac{g}{\tilde{C}_{ss}} \frac{\partial \tilde{C}_{ss}}{\partial g}
$$

and $\tilde{C}_{ss}$ is the steady state value of normalized consumption $\tilde{C}_t$,

$$
\tilde{C}_t = \frac{C_t}{m_t^{1/(1-\alpha)}}
$$

Proof. See Appendix. ■

Proposition 4 does not tell us anything about the severity of the distortions in the economy. However, we can assign values to the parameters and make numerical comparisons. Table 4 shows the different growth rates for an economy with the parameters in table 1 and 2. As it is clear from this table, for this specific parametrization, the planner sets a growth rate more than 7 percentage points above the one reached at equilibrium.

Table 4: Constrained first best vs. equilibrium

Description	Growth rate
Constrained planner	9.50%
Equilibrium	2.25%

6 Conclusion

This paper includes financial frictions into a growth model within a search framework. It shows that if financial frictions are more important for projects that are closer to the frontier of knowledge, then there could be strong impacts on growth. In particular, if the US adopted the best monitoring technology in our cross country calibration, it would increase growth to levels around 3.5%.

One limitation of this analysis is that this model assumes that banks and firms can only engage in annual contracts (we are ruling out the possibility of dynamic contracts). Moreover, we are not letting the firms accumulate assets to be less dependent of credit from banks. Therefore, the results from our numerical exercise should be interpreted as a upper bound.

Two natural ways to further improve the analysis is to introduce dynamic contracts and to allow firms to accumulate assets. I am working on the dynamic contracts extension.

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VERY INCOMPLETE

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7 Appendix

Proof of Proposition 1. Before writing the lagrangian of our problem I can simplify it. From limited liability constraints we know that x_{LL} and x_{LH} are equal to zero. On the other hand, it is obvious that the only case in which the banker might want to monitor is when the firm reports a bad outcome, hence $e_H = 0$. With this in mind we can get the following problem.

$$\xi(z, m, \Pi) \equiv \max_{\{x_{HH}, x_{HL}, e_L, k\}} \{px_{HH} + (1-p)x_{LL} - (r+\delta)k - (1-p)M(z,m)ke_L\} \tag{25}$$

st.

$$0 \leq x_{HH} \leq zk^{\alpha} \tag{26}$$

$$0 \leq x_{HL} \leq zk^{\alpha} \tag{27}$$

$$(1-e_L)zk^{\alpha} + e_L(zk^{\alpha} - x_{HL}) \leq zk^{\alpha} - x_{HH} \tag{28}$$

$$p(zk^{\alpha} - x_{HH}) = \Pi \tag{29}$$

We know that in order to receive a positive profit the bank has to set $x_{HH}, x_{HL} > 0$, so I will ignore the nonnegativity constraints as they will not be binding. The lagrangian is defined by,

$$\begin{aligned} L =& px_{HH} - (r+\delta)k - (1-p)M(z,m)ke_L + \lambda_1 \left[zk^{\alpha} - x_{HH}\right] + \lambda_2 \left[zk^{\alpha} - x_{HL}\right] \\ &+ \mu \left[zk^{\alpha} - x_{HH} - (1-e_L)zk^{\alpha} - e_L(zk^{\alpha} - x_{HL})\right] + \omega \left[\Pi - p(zk^{\alpha} - x_{HH})\right] \end{aligned}$$

To solve the problem let's assume that $e_L = 1$. The first order conditions are shown below,

$$(x_{HH}) : p - \lambda_1 - \mu + p\omega = 0 \tag{30}$$

$$(x_{HL}) : -\lambda_2 + \mu = 0 \tag{31}$$

$$(k) : -(r+\delta) - (1-p)M(z,m) + (\lambda_1 + \lambda_2)\alpha zk^{\alpha-1} - \omega\alpha pzk^{\alpha-1} = 0 \tag{32}$$

Note that since we are assuming $\Pi > 0$ we must have $x_{HH} < zk^{\alpha}$ and of course $\lambda_1 = 0$. On the other hand, note that (30) and (31) imply that $p = \mu - p\omega$. Plugging this results to (32) we get,

$$\begin{aligned} (r+\delta) + (1-p)M(z,m) &= (\mu - p\omega)\alpha zk^{\alpha-1} \\ &= p\alpha zk^{\alpha-1} \end{aligned}$$

And from the last equation I can get the value for k:

$$k = \left[\frac{\alpha pz}{r + \delta + (1-p)M(z,m)}\right]^{\frac{1}{1-\alpha}}$$

And this is the optimal k shown in the paper. With this in hand, I can get x_{HH} from (29). Further, note that any value $x_{HL} \in [x_{HH}, zk^{\alpha}]$ can be a potential solution. As it is generally done in this kind of problems, I set $x_{HH} = zk^{\alpha}$.

As a next step, I will get the bank's profit. Plugging the results in our lagrangian we get,

$$\begin{aligned}
\xi &= p\left[zk^{\alpha} - \frac{\Pi}{p}\right] - \left[r + \delta + (1-p)M(z,m)\right]k \\
&= p\left[zk^{\alpha} - \frac{\Pi}{p}\right] - \alpha p z k^{\alpha} \\
&= (1-\alpha)pzk^{\alpha} - \Pi
\end{aligned}$$

From the last equation is easy to see that there will always be a sufficiently small value of Π such that the bank is willing to lend. In other words, all firms can access to credit. Of course, setting $\xi = 0$ we can get the resulting profit function.

Lastly, it is easy to see that choosing $e_L = 0$ is not optimal. From (28) we see that in this case the bank sets $x_{HH} = 0$, which of course implies a negative profit for the bank for any $\Pi > 0$. ■

Proof of Proposition 2. First, I start showing that the aggregate variables grow at a rate $g^{(1/(1-\alpha))}$ if the variable m grows at a rate g. Let's start with the aggregate product, note that by (4) and (15),

$$y(z,m) = pzk(z,m)^{\alpha} = p\left[\frac{\alpha p z^{\frac{1}{\alpha}}}{r + \delta + (1-p)M(z,m)}\right]^{\frac{\alpha}{1-\alpha}} \tag{33}$$

Note that (33) is homogeneous of degree $1/(1-\alpha)$. Hence, I can express y in the following way,

$$\begin{aligned}
y(z,m) &= m^{\frac{1}{1-\alpha}}\tilde{y}(\tilde{z}) \\
&= m^{\frac{1}{1-\alpha}} p\left[\frac{\alpha p \tilde{z}^{\frac{1}{\alpha}}}{r + \delta + (1-p)\tilde{M}(\tilde{z})}\right]^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

Now, since the firms that decide to produce are those with $\tilde{z} \in [g, \infty)$, the aggregate product is the following,

$$Y = m^{\frac{1}{1-\alpha}} \int_{g}^{\infty} \tilde{y}(\tilde{z})\kappa\tilde{z}^{-\kappa-1}d\tilde{z} \tag{34}$$

Note that the integral in (34) is a constant that does not depend on time. Therefore, the aggregate output in the following period is equal to:

$$Y' = (gm)^{\frac{1}{1-\alpha}} \int_{g}^{\infty} \tilde{y}(\tilde{z})\kappa\tilde{z}^{-\kappa-1}d\tilde{z}$$

Hence, the growth rate of output is given by,

$$\frac{Y'}{Y} = \frac{(gm)^{\frac{1}{1-\alpha}}}{m^{\frac{1}{1-\alpha}}} = g^{\frac{1}{1-\alpha}}$$

Doing the same for the capital stock in (15),

$$\begin{aligned} k(z,m) &= m^{\frac{1}{1-\alpha}}\tilde{k}(\tilde{z}) \\ &= m^{\frac{1}{1-\alpha}}\left[\frac{\alpha p\tilde{z}}{r+\delta+(1-p)\tilde{M}(\tilde{z})}\right]^{\frac{1}{1-\alpha}} \end{aligned}$$

Thus, the equilibrium aggregate capital is defined by the following equation:

$$K = m^{\frac{1}{1-\alpha}}\int_{g}^{\infty}\tilde{k}(\tilde{z})\kappa\tilde{z}^{-\kappa-1}d\tilde{z} \tag{35}$$

And from (35) it is easy to see that aggregate capital will grow at $g^{\frac{1}{1-\alpha}}$. Now, to show that the consumption also grows at the same rate, note that market clearing implies that,

$$Y = C + K' - (1-\delta)K + (1-p)\int_{gm}^{\infty}M(z,m)kdF(z|m) \tag{36}$$

Hence, since Y and K (and of course K') grow at a rate $g^{1/(1-\alpha)}$, I only need to show that the integral in (36) grows at the same rate, I do that next. First, note again that $M(z,m)k$ is homogeneous of degree $1/(1-\alpha)$. That means that I can express the cost as:

$$M(z,m)k(z,m) = m^{\frac{1}{1-\alpha}}\tilde{M}(\tilde{z})\tilde{k}(\tilde{z})$$

Thus, doing a simple change of variables,

$$\int_{gm}^{\infty}M(z,m)k(z,m)dF(z|m) = m^{\frac{1}{1-\alpha}}\int_{g}^{\infty}\tilde{M}(\tilde{z})\tilde{k}(\tilde{z})\kappa\tilde{z}^{-\kappa-1}d\tilde{z} \tag{37}$$

And again, since the integral of the right side of (37) is constant over time, it is easy to see that the aggregate expenditure in monitoring grows at $g^{1/(1-\alpha)}$, and therefore, aggregate consumption grows at that rate. As a second step, I prove the claim regarding the interest rate. From equation (3) I get:

$$\begin{aligned} \left[g^{\frac{1}{1-\alpha}}\right]^{\sigma} &= (1+r)\beta \\ 1+r &= \frac{g^{\frac{\sigma}{1-\alpha}}}{\beta} \end{aligned} \tag{38}$$

Third, I prove our claim regarding the average spread. After some algebra using (12),(15) and (19) I get,

$$\frac{x_{HH}(z,m)}{k(z,m)} = \frac{1}{p}\left[r + \delta + (1-p)M(z,m)\right] \tag{39}$$

Now, integrating over the firms that decide to produce,

$$\overline{r_k} = \int_g^\infty \frac{1}{p}\left[r + \delta + (1-p)\tilde{M}(\tilde{z})\right] \kappa \tilde{z}^{-\kappa-1} g^\kappa d\tilde{z} \tag{40}$$

Lastly, I need to prove that my guess regarding the value of search is correct and to find the system of equations that solves for W and g. I am basically going to follow the same steps as in Perla and Tonetti (2013). Note that the indifference point will be at $\tilde{z} = g$. Hence, looking at (21), the following must hold,

$$\tilde{V}(g) = \tilde{\Pi}(g) + \frac{g^{\frac{1}{1-\alpha}}}{1+r}\tilde{V}(1) \tag{41}$$

$$= \frac{g^{\frac{1}{1-\alpha}}}{1+r} \int_g^\infty \tilde{V}\left(\frac{\tilde{z}'}{g}\right) dF_g(\tilde{z}') \tag{42}$$

In equilibrium the firms with $\tilde{z} \in [1, g]$ are going to search, so their value functions are equal to W. I apply this to (41),

$$W = \tilde{\Pi}(g) + \frac{g^{\frac{1}{1-\alpha}}}{1+r} W$$

Which implies,

$$W = \frac{\tilde{\Pi}(g)}{1 - \frac{g^{\frac{1}{1-\alpha}}}{1+r}} \tag{43}$$

From equation (42) I have the following,

$$\tilde{\Pi}(g) + \frac{g^{\frac{1}{1-\alpha}}}{1+r} W = \frac{g^{\frac{1}{1-\alpha}}}{1+r} \int_g^\infty \tilde{V}\left(\frac{\tilde{z}'}{g}\right) dF_g(\tilde{z}') \tag{44}$$

In order to find our system of equations I need to compute the right side integral of (44). Note the following,

$$\int_g^\infty \tilde{V}\left(\frac{\tilde{z}'}{g}\right) dF_g(\tilde{z}') = \int_g^\infty \tilde{V}\left(\frac{\tilde{z}'}{g}\right) \tilde{z}^{-\kappa-1}\kappa g^\kappa d\tilde{z} \tag{45}$$

$$= \int_g^{g^2} \tilde{V}\left(\frac{\tilde{z}'}{g}\right) \tilde{z}^{-\kappa-1}\kappa g^\kappa d\tilde{z} + \int_{g^2}^\infty \tilde{V}\left(\frac{\tilde{z}'}{g}\right) \tilde{z}^{-\kappa-1}\kappa g^\kappa d\tilde{z} \tag{46}$$

Let's compute the first integral of (46),

$$\int_{g}^{g^2} \tilde{V}\left(\frac{\tilde{z}'}{g}\right) \tilde{z}^{-\kappa-1}\kappa g^{\kappa} d\tilde{z} = W \int_{g}^{g^2} \tilde{z}^{-\kappa-1}\kappa g^{\kappa} d\tilde{z} \tag{47}$$

$$= W\left[1 - g^{-\kappa}\right] \tag{48}$$

Going to the second integral of (46) and noting that the firms that want to produce are those with $\tilde{z}$ higher than g,

$$\int_{g^2}^{\infty} \tilde{V}\left(\frac{\tilde{z}'}{g}\right) \tilde{z}^{-\kappa-1}\kappa g^{\kappa} d\tilde{z} = \int_{g^2}^{\infty} \left[\tilde{\Pi}\left(\frac{\tilde{z}}{g}\right) + \frac{g^{\frac{1}{1-\alpha}}}{1+r}\tilde{V}\left(\frac{\tilde{z}}{g^2}\right)\right] \kappa\tilde{z}^{-\kappa-1}g^{\kappa} d\tilde{z} \tag{49}$$

$$= \int_{g^2}^{\infty} \tilde{\Pi}\left(\frac{\tilde{z}}{g}\right) \kappa\tilde{z}^{-\kappa-1}g^{\kappa} d\tilde{z} + \frac{g^{\frac{1}{1-\alpha}}}{1+r}\int_{g^2}^{\infty} \tilde{V}\left(\frac{\tilde{z}}{g^2}\right) \kappa\tilde{z}^{-\kappa-1}g^{\kappa} d\tilde{z} \tag{50}$$

The first integral of (50) is a constant that depends on the parameters of the model. Therefore, I only need to get rid of the value function in the second integral. Using a change of variables I can get the following,

$$\frac{g^{\frac{1}{1-\alpha}}}{1+r}\int_{g^2}^{\infty} \tilde{V}\left(\frac{\tilde{z}}{g^2}\right) \kappa\tilde{z}^{-\kappa-1}g^{\kappa} d\tilde{z} = g^{-\kappa}\frac{g^{\frac{1}{1-\alpha}}}{1+r}\int_{g^2}^{\infty} \tilde{V}\left(\frac{\tilde{z}}{g^2}\right) \kappa\tilde{z}^{-\kappa-1}g^{2\kappa} d\tilde{z} \tag{51}$$

$$= g^{-\kappa}\frac{g^{\frac{1}{1-\alpha}}}{1+r}\int_{g}^{\infty} \tilde{V}\left(\frac{\omega}{g}\right) \kappa\omega^{-\kappa-1}g^{\kappa} d\omega \tag{52}$$

But then using (42) in (52) I get that,

$$\frac{g^{\frac{1}{1-\alpha}}}{1+r}\int_{g^2}^{\infty} \tilde{V}\left(\frac{\tilde{z}}{g^2}\right) \kappa\tilde{z}^{-\kappa-1}g^{\kappa} d\tilde{z} = g^{-\kappa}\left[\tilde{\Pi}(g) + \frac{g^{\frac{1}{1-\alpha}}}{1+r}W\right] \tag{53}$$

So plugging (38),(46), (48), (50) and (53) into (44) and after some algebra I get,

$$\tilde{\Pi}(g) = \frac{g^{\frac{1}{1-\alpha}}}{1+r}\int_{g^2}^{\infty} \tilde{\Pi}\left(\frac{\tilde{z}}{g}\right) \kappa\tilde{z}^{-\kappa-1}g^{\kappa} d\tilde{z} \tag{54}$$

And after a small change of variables,

$$\tilde{\Pi}(g) = \beta g^{\frac{1-\sigma}{1-\alpha}}\int_{g}^{\infty} \tilde{\Pi}\left(\omega\right) \kappa\omega^{-\kappa-1} d\omega \tag{55}$$

Hence the system (43) and (55) describe the equilibrium values of g and W, which is the result we were looking for. ■

Proof of Proposition 3. Consider equation (55). Plugging equation the profit equation (19) and after some algebra I get,

$$g^{\frac{1-\theta\alpha}{1-\alpha}} = \beta\kappa g^{\frac{1-\sigma}{1-\alpha}} \int_{g}^{\infty} \omega^{\frac{1-\theta\alpha}{1-\alpha}} \omega^{-\kappa-1} d\omega$$

Now, computing the integral,

$$g^{\frac{1-\theta\alpha}{1-\alpha}} = \frac{\beta\kappa}{\kappa - \frac{1-\alpha\theta}{1-\alpha}} g^{\frac{1-\sigma}{1-\alpha}} g^{\frac{1-\alpha\theta}{1-\alpha}-\kappa}$$

And simplifying I get,

$$g^{\frac{\kappa(1-\alpha)-1+\sigma}{1-\alpha}} = \frac{\beta\kappa}{\kappa - \frac{1-\alpha\theta}{1-\alpha}}$$

Which implies,

$$g = \left[\frac{\beta\kappa}{\kappa - \frac{1-\theta\alpha}{1-\alpha}} \right]^{\frac{1-\alpha}{(1-\alpha)\kappa+\sigma-1}}$$

■

Proof of Proposition 4. IOU

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Figure 1: Interest Margin and Economic Growth

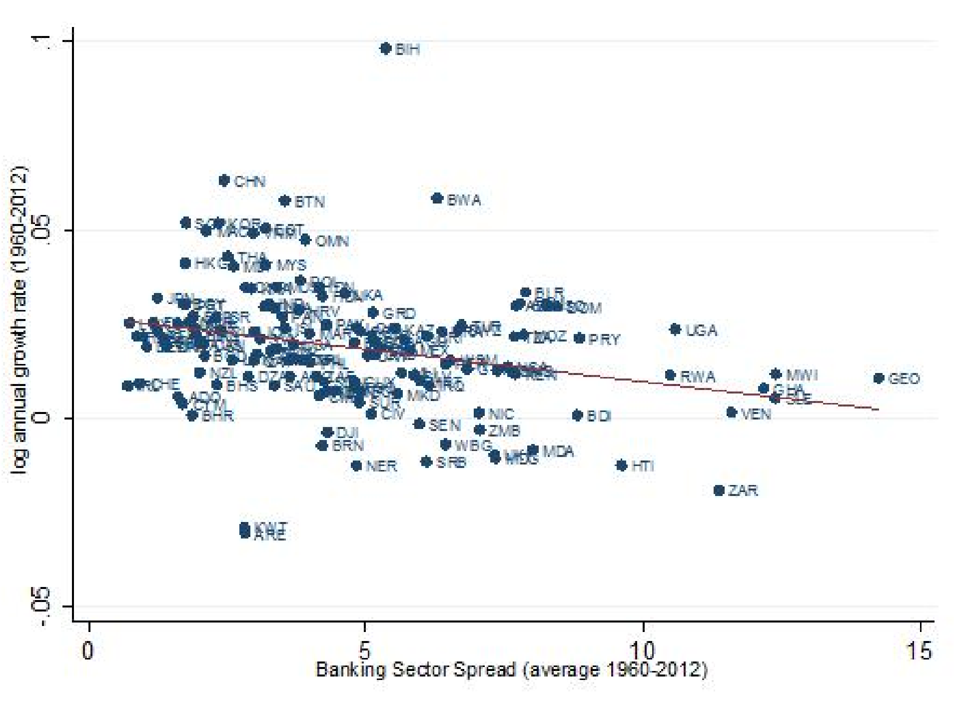

Figure 2: Timeline

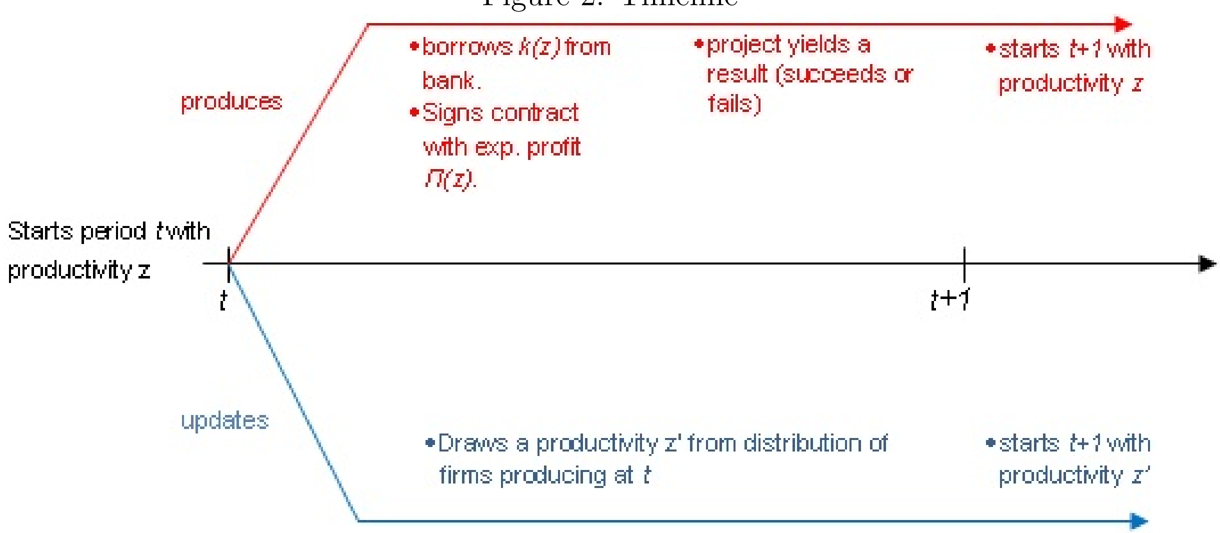

Figure 3: Results

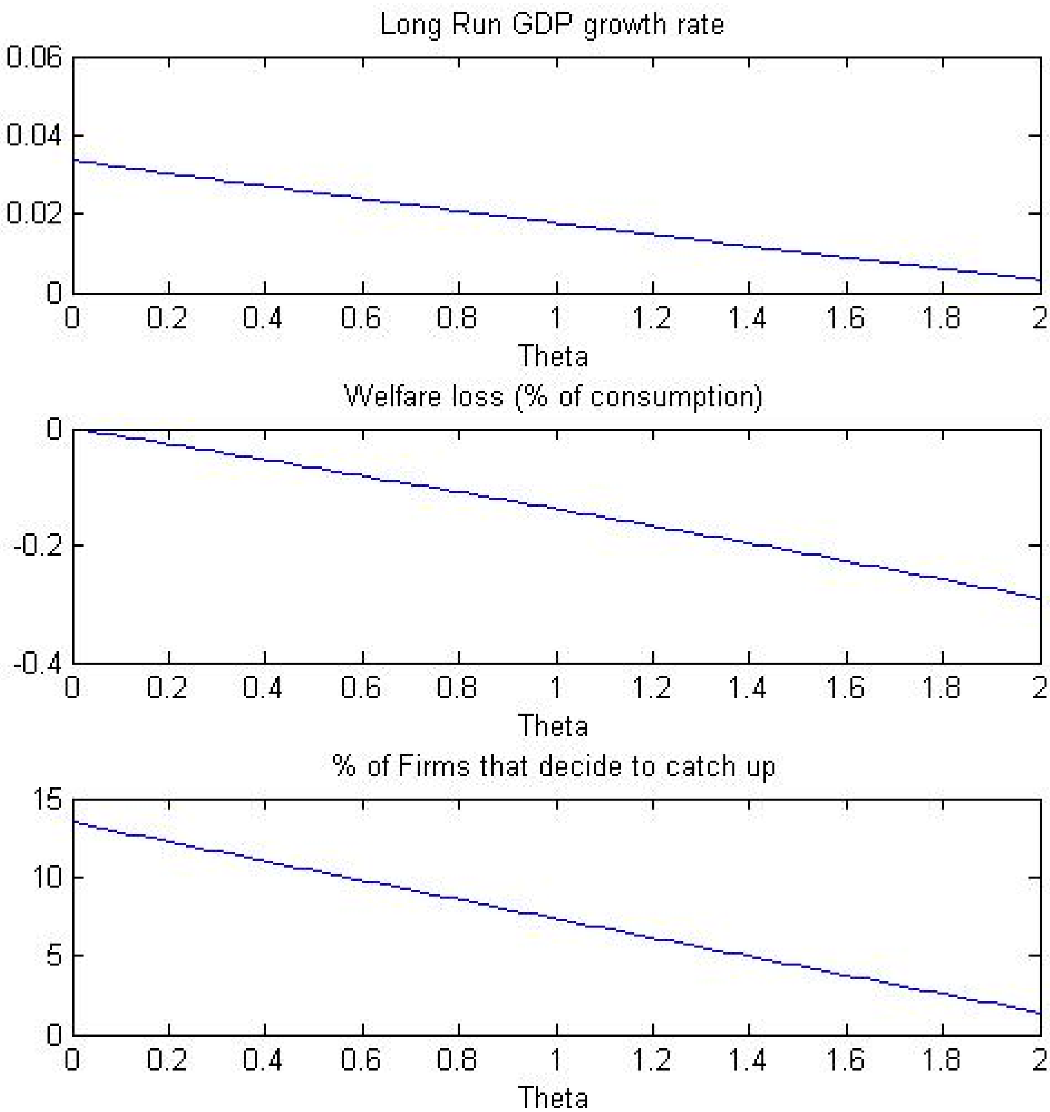

Figure 4: Model vs. Data

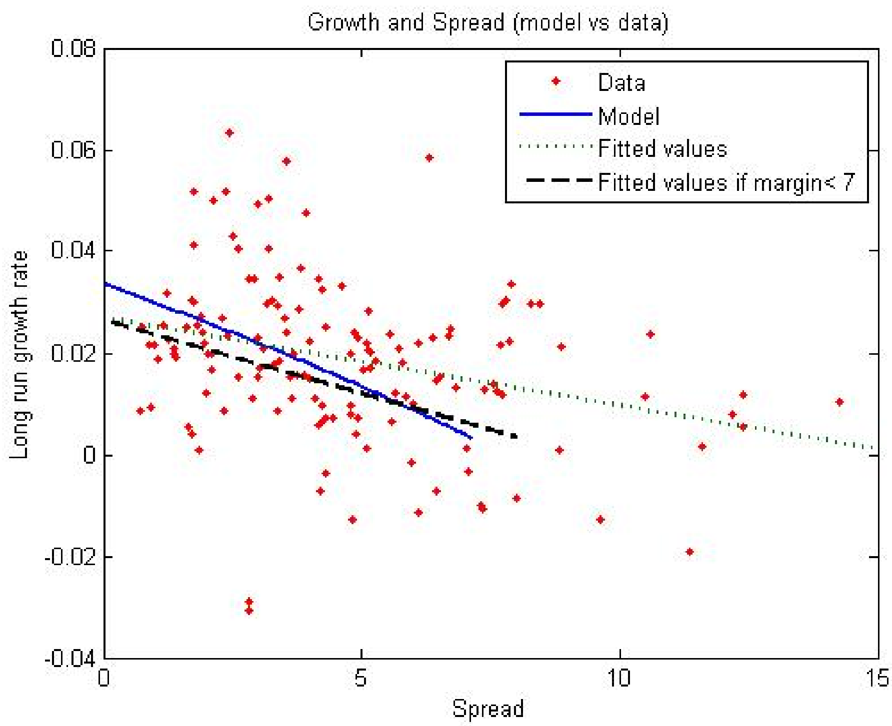

Figure 5: Cross Country θ

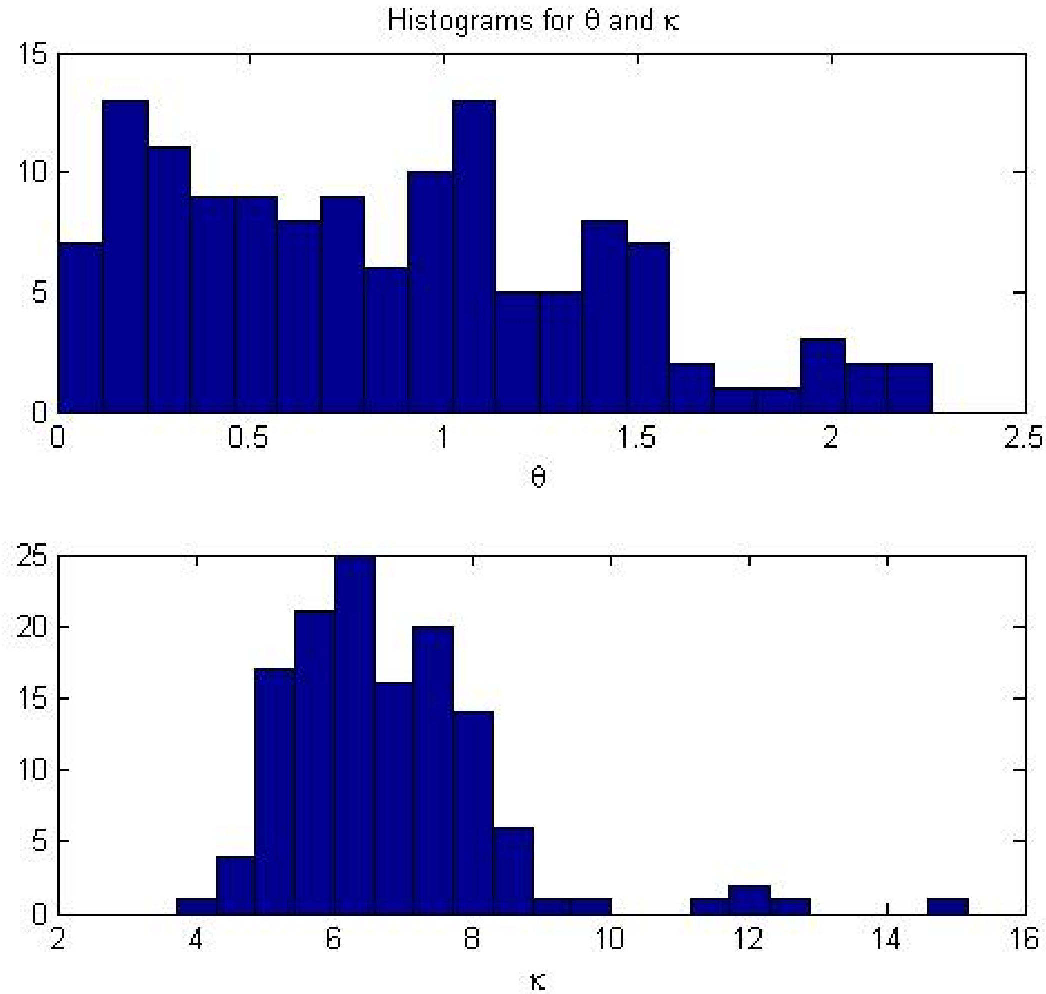

Figure 6: Cross Country θ