Parametric Recovery Methods: A Comparative Experimental Study

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Recovery of Parametric Preferences

Data: Observed choices from menus (here: linear budget set).
The Problem: recovering “approximate” parametric preferences.
Two Solutions

- Existing approaches (e.g. NLLS): minimize distance between observed and predicted choices.
  - Methodology: the model is correct, and observations represent model plus error.

- Proposal (Varian; Halevy, Persitz and Zrill): representation - minimize misspecification.
  - Misspecification: incompatibility between the ranking information encoded in choices and the ranking induced by the utility function.
  - Methodology: the model is the researcher’s approximation to choices made by the DM.
How to compare?

Based on predictive power:

- Recover using the two recovery methods.
- Run a “horse race” on different (out of sample) pairwise choices.
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Based on predictive power:
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Message: revealed preference ranking contains information about out of sample predictions.
Outline of the Talk

- Recovery using the Money Metric Index.
- Experimental design.
- Results
Observations

- Generally, researcher observes choices $x^i$ from budget sets, generated by price vector $p^i$.
- These generate a finite data set: $D = \{(p^i, x^i)_{i=1}^n\}$
Quantifying Misspecification
Quantifying Misspecification

\[ x_2 \]

\[ u \]

\[ u' \]

\[ x_1 \]

\[ I_u \]

\[ I_{u'} \]

\[ x^1 \]
v-Rationalizability

Let $v \in [0, 1]^n$

$x^i$ is $v$-directly revealed preferred to a bundle $x$, denoted by $x^i R_v^0 x$, if $v^i p^i x^i \geq p^i x$.

(strictly, denoted by $P_v^0$, if the inequality is strict).

Let $R_v$ be the transitive closure of $R_v^0$. 


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Let $R_v$ be the transitive closure of $R^0_v$.

**Definition**

Let $v \in [0, 1]^n$. A utility function $u(x)$ $v$-rationalizes $D$, if for every observed bundle $x^i$, $u(x^i) \geq u(x)$ for all $x$ such that $x^i R^0_v x$. 

Halevy & Zrill (UBC)
v-Rationalizability

If more than a single adjustment is required, one needs to aggregate v:

**Definition**

\[ f : [0, 1]^n \rightarrow [0, M], \text{ where } M \text{ is finite, is an Aggregator Function if } f(1) = 0, f(0) = M \text{ and } f(\cdot) \text{ is continuous and weakly decreasing.} \]
Money Metric Vector

Given utility function \( u(x) \) and prices \( p \), the money metric is:

\[
m(x; p, u) = \min_y py \quad \text{s.t.} \quad u(y) \geq u(x)
\]

Definition

Given a data set \( D \) and a utility function \( u(\cdot) \), the normalized money metric vector, \( v^*(D, u) \), is such that for all \( i \):

\[
v^i(D, u) = \frac{m(x^i, p^i, u)}{p^i x^i}
\]

The Money Metric Index for utility function \( u(\cdot) \) is \( f(v^*(D, u)) \).
Implications

- $f(v^*(D, u))$ may be viewed as a loss-function of using $u(\cdot)$ to represent $D$.
- $v^*(D, u)$ can be calculated observation by observation.
$\mathcal{U}^c$: the set of all continuous and locally non-satiated utility functions.

**Definition**

For a data set $D$ and an aggregator function $f(\cdot)$, let $\mathcal{U} \subseteq \mathcal{U}^c$. The *Money Metric Index of* $\mathcal{U}$ is

$$I_M(D, f, \mathcal{U}) = \inf_{u \in \mathcal{U}} f(v^*(D, u))$$
MMI vs NLLS
MMI vs NLLS: same prediction

\[ x_1^0 = x_{u'}^{0} \]
MMI vs NLLS: different predictions
Ranking Information $\equiv$ Prediction?
Operational

\[ x_2 \]

\[ x_1 \]

\[ u \]

\[ u' \]

\[ x^R \]

\[ x^S \]
Experimental Design

- We implement a 2-part experiment utilizing a graphical interface involving choice under risk
- We use the choices in Part 1 to recover parameters using NLLS and MMI
- These parameters are used to construct pairwise choices that separate the two methods
- Additionally, we focus identifying local risk attitudes (e.g. FORA)
  - Oversample moderate price ratios in Part 1
  - Oversample low-variability portfolios in Part 2
Part 1: Linear Budget Sets

- Subjects make 22 choices from linear budget sets (similar to Choi et al 2007).
- A bundle is a portfolio of contingent assets with two equally probable states.
- Budget lines are chosen so as to:
  - provide a powerful test of rationality (GARP)
  - identify local risk attitude in the neighborhood of certainty
  - identify possible wealth effects
Parametric Specification

Disappointment Aversion (in 2-states similar to RDU):

\[ u(x^i) = \gamma w(\max \{ x_1^i, x_2^i \}) + (1 - \gamma) w(\min \{ x_1^i, x_2^i \}) \]

where

\[ \gamma = \frac{1}{2 + \beta} \]
\[ \beta > -1 \]

and

\[ w(z) = \frac{z^{1-\rho}}{1-\rho} \quad (CRRA) \]
Indifference Curves

(a) Disappointment Aversion - \( \beta > 0 \)

(b) Elation Loving - \( \beta < 0 \)
For each individual, in the background and without the subject's knowledge:

- we recover parameters using the MMI and NLLS using quadratic aggregators.
- construct pairwise choice sets designed to separate the two sets of parameters
- these choice sets are constructed using a computer algorithm
Part 2: Pairwise Choice

- Subjects make 9 pairwise choices
- Each choice is between a risky portfolio and a safe (certain) portfolio
- By construction, for all choice problems, one of the portfolios is preferred by one set of parameters
The Algorithm
Results

- For each subject we report the number of times (out of 9) the MMI correctly predicts pairwise choice.
- We report results at both the individual and aggregate level.
## Unrefined Results - Aggregate

<table>
<thead>
<tr>
<th></th>
<th># of Observations</th>
<th>Correct Predictions by MMI (%)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>1863</td>
<td>1012 (54.3%)</td>
<td>0.0001</td>
</tr>
<tr>
<td>Low-variability</td>
<td>1242</td>
<td>669 (53.9%)</td>
<td>0.0035</td>
</tr>
<tr>
<td>High-variability</td>
<td>621</td>
<td>343 (55.2%)</td>
<td>0.0051</td>
</tr>
</tbody>
</table>

**Table:** Aggregate results - All subjects (n=207)
### Table: Individual Results - All Subjects (n=207)

<table>
<thead>
<tr>
<th>$X \geq 7$</th>
<th>$X \leq 2$</th>
<th>$\Pr(X \geq 7)$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>42</td>
<td>0.09</td>
<td>0.0204</td>
</tr>
<tr>
<td>$X \geq 8$</td>
<td>$X \leq 1$</td>
<td>$\Pr(X \geq 8)$</td>
<td>p-value</td>
</tr>
<tr>
<td>43</td>
<td>24</td>
<td>0.02</td>
<td>0.0136</td>
</tr>
<tr>
<td>$X \geq 9$</td>
<td>$X \leq 0$</td>
<td>$\Pr(X \geq 9)$</td>
<td>p-value</td>
</tr>
<tr>
<td>28</td>
<td>16</td>
<td>0.002</td>
<td>0.0481</td>
</tr>
</tbody>
</table>
We use revealed preference to test for rationality in Part 1. Exclude subjects with Afriat Efficiency Index less than 0.9, i.e. choices imply more than 10% efficiency loss due to violations of rationality. 194 of 207 subjects survive this refinement (92 subjects pass GARP).
Different Parameters

- There is no sense in comparing MMI and NLLS when they recover similar values.
- Not obvious how to determine when there is a difference.
- Exclude when:
  - $\max\{|\beta_{MMI} - \beta_{NLLS}|, |\rho_{MMI} - \rho_{NLLS}|\} < 0.1$.
  - $\beta$ is unidentified (almost all portfolio close to the 45-degree line).
- 142 out of 194 subjects survive this refinement.
## Refined Results - Aggregate

<table>
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<th>Correct Predictions by MMI (%)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>1278</td>
<td>730 (57.1%)</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Low-variability</td>
<td>852</td>
<td>487 (57.2%)</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>High-variability</td>
<td>426</td>
<td>243 (57.0%)</td>
<td>0.0021</td>
</tr>
</tbody>
</table>

**Table**: Aggregate Results - Refined Subjects (n=142)
Refined Results - Individual

<table>
<thead>
<tr>
<th>X \geq 7</th>
<th>X \leq 2</th>
<th>Pr(X \geq 7)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>22</td>
<td>0.09</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X \geq 8</th>
<th>X \leq 1</th>
<th>Pr(X \geq 8)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>11</td>
<td>0.02</td>
<td>0.0014</td>
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<table>
<thead>
<tr>
<th>X \geq 9</th>
<th>X \leq 0</th>
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<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>6</td>
<td>0.002</td>
<td>0.0047</td>
</tr>
</tbody>
</table>
## Recovered Parameters

<table>
<thead>
<tr>
<th></th>
<th>MMI</th>
<th>NLLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$</td>
<td>$\rho$</td>
</tr>
<tr>
<td>All</td>
<td>0.4130</td>
<td>0.3854</td>
</tr>
<tr>
<td>AEI &gt; 0.90</td>
<td>0.4375</td>
<td>0.3876</td>
</tr>
<tr>
<td>GARP</td>
<td>0.4242</td>
<td>0.3061</td>
</tr>
<tr>
<td>Choi et al</td>
<td>0.333</td>
<td>0.356</td>
</tr>
</tbody>
</table>

**Table:** Median Values
First-Order Risk Aversion

- We also observe First-Order Risk Aversion directly in subject’s pairwise choices.
- Of the 207, with respect to the 6 low-variability portfolios:
  - 156 choose the safe option at least once
  - 51 choose the safe option for all 6 portfolios
Conclusion

- Misspecification of preferences is virtually assured when estimating simple functional forms.
- HPZ proposed a loss-function that is based on minimizing the incompatibility between the ranking information encoded in choices and the ranking induced by the utility function.
- Recovery that relies on revealed preference information $\Rightarrow$ better prediction!
- Importance of First-Order Risk Aversion in portfolio choice problems.