A Fraudulent Expert and Short-Lived Customers*

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Abstract

A market where short-lived customers interact with a long-lived expert is considered. An expert privately observes whether or not a particular treatment is necessary for his customers and has an incentive to recommend the treatment even if it is unnecessary. Customers imperfectly observe the expert’s past actions. Truthful reporting at all times yields the expert his best equilibrium payoff when the expert is known to be opportunist (i.e., rational in the usual sense). If the customers believe that the expert might be an honest type, who always reports truthfully, then the expert can build his reputation for honesty, so then he defrauds his customers to achieve a higher payoff. Deception during an unbounded length of time is a zero-probability event in equilibrium. However, it is a probability one event (in some of the equilibrium) when the expert’s customer is also a long-lived agent. (JEL C72, C73, D82, D83)

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1. Introduction

Many forms of consulting and advisory, medical, or repair services are prime examples of what is known as a credence or an experience good in the economics literature. Generally speaking, these goods have the characteristics that customers can observe the utility they derive from the good *ex post* (i.e., upon consumption) but cannot be sure about the extent of the good they actually need *ex ante*. Therefore, sellers act as experts who determine the customers’ needs by performing a diagnosis. They can then provide the right quality and charge for it or exploit the information asymmetry by defrauding the customer.

Deception or mistreatment is an important source of inefficiency that occurs in experience and credence goods markets, and customers’ concerns about being cheated by experts are confirmed by empirical studies. Emons (1997) cites a Swiss study reporting that an average person’s probability of receiving one of seven major surgical interventions is one-third above that of a physician or a member of a physician’s family. In the late 1970s, the Department of Transportation estimated that 53% of auto-repair charges represented unnecessary repairs (see Wolinsky, 1993 and 1995). More recently, a field experiment by Schneider (2012) shows that completely unnecessary repairs were present in 27% of the cases, and serious undertreatment occurred in 77% of the cases. He also estimates that agency problems in the U.S. auto-repair market generate a welfare loss of approximately $8.2 billion, or 22% of industry revenue. Levitt and Syverson (2012) claims that real estate agents have an incentive to convince clients to sell their houses too cheaply and too quickly. He reports that for two comparable houses, one owned by a real estate agent and the other owned by a client of the real estate agent, the home of the real estate agent will stay on the market for a longer period (an extra 9.5 days) and sell for a higher price (approximately 3.7%), and the greater the informational advantage of the real estate agent, the larger these two differences are. Considerable evidence that exists in the health-care industry also indicates that monetary incentives matter for the provision of credence goods. Gruber, Kim, and Mayzlin (1999), for example, show that the frequencies of cesarean deliveries compared with normal childbirths react to the fee differentials of health insurance programs.

1Nelson (1970) first made the distinction between a search good for which quality is evident prior to purchase and an experience good for which quality is known only after consumption. The notion of a credence good for which quality may never be known is proposed by Darby and Karni (1973).

2There are two strands of literature on the credence goods (Dulleck, Kerschbamer, and Sutter, 2011). One strand takes the abovementioned characteristics—customers do not know what they need, but they observe the utility from what they consume. The other strand assumes that customers know what they need but observe neither what they consume nor the utility derived from what they consume (e.g., whether food has been produced organically or not).

3See the surveys in McGuire (2000) and Gaynor and Vogt (2000).
In this study, I consider markets for expert advice (e.g., experience and credence goods markets) and address the question of whether the life span of the relationship between an expert seller and his customers makes the seller more or less prone to fraudulent behavior. Considering this question, I also investigate the roles of monitoring, reputation, and trust on sellers’ tendency to defraud.

This question is important because the pursuit of “trust-based” long-run relationship with customers is a dominant theme in business management and marketing today. However, it is not obvious if “trust-based” long-run relationships necessarily hinder sellers’ incentives to engage in fraudulent activity. In fact, this study shows that an expert seller can deceive his customers for an unbounded length of time when he is engaged in a (trust-based) long-run relationship with them.

Meanwhile, it is not always possible for a seller to construct a long-run relationship with his customers. Some customers are infrequent shoppers, and some goods are simply not suited for repetitive purchases. People who buy a house do not buy a new one every year—and if they do, they are probably not going back to the same real estate agent. When the customers are “short-lived” agents, the sellers’ reputation (experience) appears to have a critical role on customers’ decision making. Therefore, the question “Is an expert seller more prone to fraudulent behavior when his customers are ‘short-lived’ or ‘myopic’ agents?” is equally important.

To address the abovementioned question, I study an infinite-horizon game where a long-lived expert seller repeatedly plays a simple sender-receiver game against a succession of agents (short-lived customers), each of whom plays the game once. At each stage, a customer (e.g., a car owner) seeks a service of the expert (a mechanic) who can correctly diagnose what is necessary for the customer and offer a particular treatment. Conditional on observing the seller’s advice, each customer either approves or rejects the treatment. The customers’ payoff of rejecting the treatment is normalized to 0. Approving the treatment yields customers a negative payoff if the treatment is unnecessary and a positive payoff if the treatment is necessary. Customers are uncertain whether the expert’s treatment is necessary or not and cannot verify the seller’s advice. The customers’ prior beliefs about the necessity of the treatment is such that their ex ante payoff of approving the treatment is negative. That is, the customers may approve the treatment only if they trust the expert that there is some truth in his advice. Therefore, the long-lived expert is facing a trade-off between manipulating his customers’ decisions through his advice for short-term gains and reporting truthfully to sustain the credibility of his future advice.

The results indicate that an equilibrium of this repeated sender-receiver game exists
in which the expert’s advice is “influential” (i.e., informative and valuable). Deception is consistent with equilibrium. That is, an equilibrium of this repeated sender-receiver game exists, where the expert offers an unnecessary treatment to a customer and the customer approves the treatment. However, if the short-lived customers are certain that the expert is an opportunist type (i.e., rational in the usual sense), then deception does not improve the expert’s payoff. That is, the expert can achieve his best equilibrium payoff by truthfully reporting at all stages. The reason for this is that the short-lived customers are so alert against the expert’s advice that they never trust him unless he tells the truth with a sufficiently high probability. Because the expert must play a mixed strategy (i.e., tell the truth with a positive probability), if he wants to deceive his customers, his payoff from deception can be no more than his payoff of telling the truth.

However, if the customers believe that the expert might be an honest type, who always reports truthfully, then the expert would achieve a higher payoff by deceiving his customers. Therefore, the ability of building reputation for honesty makes the expert seller more prone to fraudulent behavior. Brown and Minor (2012) empirically support this prediction and show that more experienced experts are significantly more likely to mislead their customers. The good news is that the expert’s deceits are not limitless. The expected number of misleading advice an expert can give in an equilibrium is bounded by a finite number $1/\beta$, where $\beta \in [0, 1]$ is the rate at which a customer’s experience with the seller becomes public information.\[4\]

For a benchmark result, I study the repeated sender-receiver game with two long-lived agents. An equilibrium of this game exists in which the expert seller deceives his long-lived customer for an unlimited period. Therefore, the expert seller might be more deceitful when he is engaged in a long-term relationship with his customers. Deceiving a long-lived customer for a very long time is consistent with equilibrium because the expert can credibly promise his customer that he will report truthfully for a sufficiently long period in the future if she tolerates one stage of deception today. The expert seller’s long-term promises have no benefit to short-lived customers. Thus, the expert cannot exploit his short-lived customers as much as he exploits his long-lived customer.

Section 2 explains the details of the sender-receiver game and provides the equilibrium predictions of this stage game. Section 3 discusses the repeated sender-receiver game with short-lived receivers and presents the main results of the study. Section 4 considers the repeated sender-receiver game where both the seller and his customer are long-lived agents. Finally, Section 5 concludes and discusses the related literature.

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\[4\] Put differently, $\beta$ measures how perfect the customers’ monitoring technology is. Thus, $\beta = 1$ indicates the perfect public monitoring technology.
2. The Sender-Receiver Stage Game

In this section, I will introduce and examine the sender-receiver game where an expert (he) and a receiver (she) interact only once. Section 3 investigates the infinitely repeated sender-receiver game. The receiver (e.g., a car owner) seeks service of an expert (a mechanic) who provides a particular treatment. The expert can correctly diagnose what is necessary for the receiver, but may have an incentive to mislead her. On the other hand, the receiver is unsure whether the expert’s particular treatment is necessary, but cannot verify the expert’s advice before accepting his treatment.

The timing of the game is as follows: At the beginning, the nature determines whether the treatment is “necessary” \( n \) or “unnecessary” \( u \), and so the true state is \( s \in S = \{ n, u \} \). The treatment is unnecessary with probability \( \pi \in (0, 1) \). The expert observes the true state and then sends an unverifiable message to the receiver \( m \in M = S \). After observing the expert’s message, the receiver who does not know the true state either approves \( a \) or rejects \( r \) the treatment. Regardless of the true state, the expert’s payoff is \( v_e > 0 \) if the receiver approves the treatment and 0 otherwise. The receiver’s payoff of approving the treatment is positive if the treatment is necessary, but negative if the treatment is unnecessary. Her payoff of rejecting the treatment is normalized to 0. The payoffs are summarized as follows:

\[
\begin{array}{c|cc}
  & a & r \\
 s = u & v_e, -v_u & 0, 0 \\
 s = n & v_e, v_n & 0, 0 \\
\end{array}
\]

where the real numbers \( v_e, v_u, \) and \( v_n \) are all strictly positive. I suppose that the parameters satisfy

\[-\pi v_u + (1 - \pi) v_n < 0\]  (1)

so that the receiver’s expected return from the treatment is negative. Thus, ex ante, the receiver prefers to reject the treatment.

Furthermore, the expert is one of two types: honest and opportunist. The opportunist expert is a rational player in the usual sense. That is, he chooses his message, given his beliefs about the receiver’s play, to maximize his expected payoff. However, the honest expert always tells the truth. The expert knows his type, and let \( \mu \in [0, 1) \) denote the probability that the expert is honest (i.e., \( \mu \) is the expert’s initial reputation). Call this sender-receiver game where all parameters are common knowledge \( G \).

Let \( \sigma_e(s) \in [0, 1] \) denote the probability that the expert sends the message \( u \) when
he observes state $s$. Therefore, $1 - \sigma_e(s)$ is the probability that the expert sends the message $n$ in state $s$. Given the expert’s strategies $\sigma_e(u)$ and $\sigma_e(n)$, the receiver updates her belief about the true state according to the Bayes’ rule. Let $P(s|m)$ indicate the receiver’s posterior probability that the true state is $s$ conditional on the event that the expert sends the message $m$. Thus,

$$P(u|n) = \frac{\pi(1-\mu)[1-\sigma_e(u)]}{\pi(1-\mu)[1-\sigma_e(u)] + (1-\pi)(1-\mu)[1-\sigma_e(u)]}.$$

The receiver’s mixed strategy $\sigma_r(m)$ is a function of the message $m \in S$ she receives. Therefore, $\sigma_r(m) \in [0,1]$ indicates the probability that the receiver plays $a$ when she observes message $m$. A strategy profile and a belief structure constitute a Perfect Bayesian Equilibrium (or simply equilibrium) if (1) each player’s strategy specifies optimal actions, given his/her beliefs and the strategies of the other player, (2) given the strategy profile, the beliefs are consistent with Bayes’ rule whenever possible.

**Definition 1.** An equilibrium is **fully revealing** if the (opportunist) expert truthfully reports the state, that is, he sends the message $m$ if and only if the true state is $m$. An equilibrium is **influential** if the receiver approves the treatment with a positive probability. An equilibrium is **babbling** if the (opportunist) expert’s strategy is independent of the true state and the receiver’s strategy is independent of the expert’s message.

Given these definitions, the optimality of equilibrium automatically implies the following claims that are represented in the next remark with no formal proof.

**Remark 1.** A fully-revealing equilibrium is influential. However, an influential equilibrium is not necessarily fully revealing. In fact, deception can occur in an equilibrium that is influential but not fully revealing.

**Proposition 1.** There does not exist a fully-revealing equilibrium of the sender-receiver game $G$. Moreover, the expert’s deception is consistent with equilibrium if and only if the expert’s reputation for honesty $\mu$ is higher than

$$\mu^* \equiv 1 - \frac{(1 - \pi)v_u}{\pi v_u}.$$

In particular, there exists an influential equilibrium, where the expert is deceitful, if and only if $\mu \geq \mu^*$.

In equilibrium where the expert is deceitful, the opportunist expert always sends the message $n$ and the receiver approves the treatment only when she observes the message
n. Therefore, by expert’s deception (fraud) we formally mean that the receiver approves the treatment when she observes the message n and that the expert sends the message n although the true state is u.

It is rather easy to see why there is no fully revealing equilibrium. Suppose that there is an equilibrium in which the expert sends the message m if and only if the true state is m. Given the expert’s strategy, the receiver’s best response is to approve the treatment only when she observes the message n. However, given the receiver’s strategy, the best response for the expert is to lie and to send the message n, not u, when the true state is in fact u. The threshold $\mu^*$ indicates the minimum level of trust the expert needs to possess in order to deceive the receiver. That is, when the expert’s reputation is higher than or equal to $\mu^*$, there is an equilibrium of the sender-receiver game G in which the receiver approves the treatment whenever she observes the message n. It is important to note that $\mu^*$ depends solely on the receiver’s expected return from the treatment.

There does not exist a babbling equilibrium when $\mu \geq \mu^*$ because the receiver prefers to accept the treatment whenever she observes the message n, and so, the receiver’s equilibrium strategy will certainly depend on the message she receives. Therefore, the influential equilibrium where the expert is deceitful essentially is the unique equilibrium of the sender-receiver game G when $\mu \geq \mu^*$. However, a babbling equilibrium, where the expert always sends the message n and the receiver always rejects the treatment, exists if the expert’s reputation for honesty is lower than the threshold level $\mu^*$.

\textbf{Remark 2.} A babbling equilibrium exists if and only if $\mu < \mu^*$.

3. The Repeated Sender-Receiver Game with Short-Lived Receivers

This section studies the repeated sender-receiver game with short-lived receivers. The timing of the repeated sender-receiver game is as follows. The nature moves first and determines the expert’s type. The expert is either honest (with probability $\mu \in [0, 1]$) or opportunist. The type of the expert is fixed throughout the game. Only the expert knows his type. At each stage $t \in \{0, 1, \ldots\}$ the expert and the receiver play the stage game G that was described in Section 2. At the beginning of each stage, the nature determines the true state $s \in S$, where $\pi \in (0, 1)$ is the probability that the state is u. The expert observes the state and sends his message $m \in M$. After observing the expert’s message, the receiver decides whether to approve or reject the treatment. At the end of each stage, the expert and the receiver obtain their stage game payoffs. The receiver cannot learn the true state if she rejects the treatment. The payoff structure of the stage game G was already given in Section 2.
The expert is a long-lived agent with a discount factor $\delta < 1$. Thus, the opportunist expert’s objective is to maximize his discounted lifetime payoffs. The receiver, on the other hand, is an infinite sequence of different short-lived agents who play the stage game with the expert only once (Fudenberg and Levine 1989). Therefore, each short-lived receiver’s objective is to maximize her expected payoff in the stage game she plays.

The expert can perfectly observe the entire history of the repeated sender-receiver game. The short-lived agents (i.e., potential receivers) all observe the same public signal $y_t \in Y$ at the end of stage $t$. Suppose that the public information at the start of stage $t$ is $h^t = (y^0, \ldots, y^{t-1})$. For any $t \geq 0$, $y^t \in Y = \{\emptyset, u_r(a^t|s^t)\}$. By $y^t = \emptyset$, I mean that the receivers observe no information about the stage $t$ play. One can interpret this case as the short-lived agent who plays the game with the expert in stage $t$ does not share her experience with the other short-lived agents. On the other hand, $u_r(a^t|s^t)$ is the $t$th-stage receiver’s payoff realization, which is a function of her action $a^t$ and the true state $s^t$ in stage $t$.

In particular, for any $t$, $y^t = \emptyset$ with probability $1 - \beta$ and

$$y^t = u_r(a^t|s^t) = \begin{cases} -v_u, & \text{if } (a^t, s^t) = (a, u) \\ v_n, & \text{if } (a^t, s^t) = (a, n) \\ 0, & \text{otherwise.} \end{cases}$$

with probability $\beta$. The term $\beta \in [0, 1]$ is the rate at which a receiver’s experience with the sender becomes public information. Higher values of $\beta$ ensures that the receivers will be better informed about the expert’s past play. Call this repeated sender-receiver game where all parameters are common knowledge $G^\infty$.

Let $\mathcal{H}_e = \bigcup_{t=0}^{\infty} (A \times S \times M)^t$ denote the set of histories for the expert. Therefore, a behavioral strategy of the opportunist expert is $\sigma_e : \mathcal{H}_e \times S \rightarrow \Delta(M)$. Given any history $h^t$ (possibly a null history $h^0$), $\sigma_e(h^t, s)(m)$ denotes the probability that the opportunist expert sends message $m$ given that he observes state $s$ after history $h^t$. The strategy of the honest expert is simple: he reports the true state at any stage. A behavioral strategy for the receiver is $\sigma_r : \mathcal{H}_r \times M \rightarrow \Delta(A)$ where $\mathcal{H}_r = \bigcup_{t=0}^{\infty} Y^t$. Let $\sigma_r(h^t, m)(a)$ denote the probability that the receiver approves the treatment given that she observes message $m$ after history $h^t$. If $\{a^t\}_{t=0}^{\infty}$ is the sequence of actions taken by the receiver and if $\{s^t\}_{t=0}^{\infty}$ is the sequence of state realizations throughout the game, then the expert’s payoff is $\sum_{t=0}^{\infty} \delta^t u_e(a^t|s^t)$. The payoff of the receiver who enters the game at stage $t$ is simply $u_r(a^t|s^t)$.

**Definition 2.** An equilibrium of the repeated sender-receiver game $G^\infty$ is **fully revealing**
if the (opportunist) expert reports truthfully at all stages on the equilibrium path. It is *babbling* if the (opportunist) expert’s strategies are independent of the true state and the receivers’ strategies are independent of the expert’s message at all stages on the equilibrium path. Finally, a fully-revealing equilibrium of the game $G^\infty$ is *influential* if the receivers approve the project whenever they observe the message $n$.

**The Main Results**

Suppose for now that $\beta = 0$, that is, the short-lived receivers cannot observe the history of the repeated sender-receiver game $G^\infty$. This case resembles situations where the short-lived agents can never learn the benefit of the treatment as is true for some credence goods. There cannot exist a fully revealing equilibrium of the repeated sender-receiver game because the short-lived receivers cannot coordinate on the expert’s past play. In addition, the expert cannot build up or loose his reputation in the game. Therefore, if the expert’s initial reputation for honesty is small (i.e., $\mu < \mu^*$), then there exists no equilibrium in which a receiver approves the treatment. On the other hand, if $\mu \geq \mu^*$, then there exists a unique equilibrium in which each receiver approves the treatment and the expert deceives the receivers and sends message $n$ at all times. For the rest of this section, I will investigate the case where the receivers can observe the expert’s past play, that is $\beta \in (0, 1]$.

**Proposition 2.** For any $\mu \in [0, 1)$ and $\beta \in (0, 1]$ there is some $\delta_{\beta} \in (0, 1)$ such that for all $\delta > \delta_{\beta}$ there exists a fully-revealing and influential equilibrium of the game $G^\infty$. Furthermore, when $\mu = 0$

1. deception is consistent with equilibrium, but the fully-revealing equilibrium, yielding payoff of $v_e' = \frac{(1-\pi)n\mu}{1-\beta}$, is the expert’s best equilibrium, and

2. the expert’s worst equilibrium is a babbling equilibrium with payoff of 0.

For any $\beta \in (0, 1]$, the repetition of the babbling equilibrium of the stage game, where the opportunist expert sends the message $n$ regardless of the true state and the receiver rejects the treatment independent of the message she observes, is the equilibrium of the repeated game $G^\infty$ given that $\mu < \mu^*$. In this babbling equilibrium, the expert’s and each receiver’s payoffs are 0. However, there exist other equilibrium where the expert can achieve higher payoffs. If the expert is sufficiently patient, then mutual trust between the short-lived receivers and the opportunist expert supports an equilibrium where the expert truthfully reports at all stages. A punishment strategy that supports the fully-revealing
equilibrium is simple: if the expert deviates and deceives a receiver (i.e., sends message \(n\) when the true state is \(u\)), and if this deviation is observed by the short-lived receivers, then the expert and all the subsequent receivers play their babbling equilibrium strategies for the rest of the game.

Deception is consistent with equilibrium in the repeated sender-receiver game \(G^\infty\) if the expert is known to be the opportunist type. However, in this case, deception has no additional benefit to the expert. In fact, the expert achieves his highest payoff in the fully revealing equilibrium by simply being truthful at all stages. It is important to note that fully revealing equilibrium is the best equilibrium (ex post) for each receiver. Therefore, in any equilibrium where the expert’s payoff is strictly higher than \(v^f_e\), the expert will deceive some of the receivers, and thus, some receivers get negative payoffs (ex post). Thus, there is a positive relationship between the expert’s equilibrium payoff (as long as it is higher than \(v^f_e\)) and the number of deception (misleading advice giving).

A short-lived receiver does not care about the expert’s future behavior. For this reason, “full deception” is not consistent with equilibrium if \(\mu = 0\). Full deception occurs when the opportunist expert sends message \(n\) with certainty after observing state \(u\). More formally, after any history \(h^{t-1}\), I call that the expert fully deceives the receiver in stage \(t\) if \(\sigma_e(h^{t-1},u)(n) = 1\) and partially deceives the receiver in stage \(t\) if \(\sigma_e(h^{t-1},u)(n) \in (0, 1)\).

In equilibrium, stage \(t\) receiver will approve the treatment after observing message \(n\) only if her posterior belief that the true state is \(n\) is high enough so that her expected payoff of approving the treatment is no less than 0, which is her payoff of rejecting the treatment. Therefore, the expert who is known to be the opportunist type can deceive a short lived receiver in stage \(t\) if the expert’s probability of lying in that stage is sufficiently small when the true state is \(u\). However, a behavioral strategy \(\sigma_e(h^{t-1},u)(n) \in (0, 1)\) is consistent with equilibrium if and only if the expert’s continuation payoff of sending message \(u\) and \(n\) after observing the history \((h^{t-1},u)\) are the same. This is why partial deception will not give the expert a payoff higher than what he would achieve if he had been truthful. As a result, deception improves the expert’s payoff only if the expert can fully deceive the receivers.

The next result (i.e., Proposition 3) proves that the expert can do better than being truthful when there is some uncertainty regarding the expert’s type (i.e., \(\mu\) is positive). The threshold reputation level \(\mu^* = 1 - \frac{(1-\pi)v_u}{\pi v_n}\) plays a significant role. If \(\mu \geq \mu^*\), then the expert can deceive the receivers without building further reputation for honesty. However, if \(\mu < \mu^*\), then the expert first needs to build up his reputation to be able to

\[5\text{In particular, we must have } \sigma_e(h^{t-1},u)(n) \leq \frac{v_u(1-\pi)}{v_u(1-\pi) + v_n}\]
deceive the receivers.

If the receivers’ conjecture is such that the expert strictly prefers to tell the truth at stage $t$, then observing the expert telling the truth at that stage does not change the receivers’ belief about the expert’s actual type (i.e., the expert’s reputation); he simply does what he was expected to do. However, observing the expert telling the truth even though he strictly prefers to lie changes the receivers’ belief about the type of the expert. But this observation also proves that the receivers’ conjecture was wrong, and “equilibrium” dictates that the receivers must have right conjectures to begin with. Therefore, in equilibrium, the expert can build his reputation for honesty if the receivers have the right conjecture that the expert has incentives to lie and to tell the truth (i.e., he is indifferent between lying and telling the truth). Thus, if he lies, then he chooses to materialize his short-term incentives as expected. If instead he tells the truth, then he chooses to postpone his short-term gains for something higher in return, which is a higher reputation for honesty.

**Lemma 1.** Suppose that $\mu > 0$ and $\beta \in (0, 1]$. In equilibrium, the shortest time (i.e., the smallest number of stages) that is required for the expert to build up his reputation to $\mu^*$—while the receivers prefer to approve the treatment conditional on observing the message $n$—is

$$N_G = \begin{cases} 0 & \text{if } \mu \geq \mu^* \\ K^* & \text{otherwise}, \end{cases}$$

where $K^*$ is the smallest positive integer satisfying

$$(\mu^*)^{K^*+1} \leq \mu, \text{ that is}$$

$$K^* = \min \left\{ k \in \mathbb{Z}^+ \left| \frac{\ln \mu}{\ln \mu^*} - 1 \leq k \right. \right\}.$$

If $\mu \geq \mu^*$, then the expert does not need to build up his reputation, and thus $N_G$ is simply $0$. The interesting case is when $\mu < \mu^*$. Because our ultimate purpose (in Proposition 3) is to find a strategy in which the expert’s payoff is the highest, this strategy of the expert should dictate him to tell the truth (1) when the true state is $n$, (2) with a sufficiently low probability when the true state is $u$ so that his reputation is updated quickly, and (3) with a sufficiently high probability when the true state is $u$ so that the receivers approve the treatment when they observe the message $n$. The reason why the first condition should hold is obvious. Because the expert discounts time, he prefers to play a strategy in which he can build his reputation as fast as he can, and thus, he can start deceiving the receivers as early as he can. Given that the expert tells the truth at the first stage with probability $\sigma_e \left( \equiv \sigma_e(\emptyset, u)(u) \right)$, his reputation following a history $h^1$, where the state is $u$ in the first stage and the expert tells the truth and sends the message
u, is \( \mu_1 = \frac{\mu}{\mu + (1-\mu)\sigma_e} \) according to the Bayes’ rule. The expert can deceive the receiver in the second stage only if his updated reputation is higher than \( \mu^* \) (i.e., \( \mu_1 \geq \mu^* \)). Thus, the expert can update his reputation to the required level \( \mu^* \) in only one stage if the following holds:

\[
\sigma_e \leq \frac{(1 - \mu^*)\mu}{\mu^*(1 - \mu)}.
\]  

(2)

As for the third condition, if the expert’s strategy \( \sigma_e \) is very low (so the expert lies with a very high probability when the true state is \( u \)), then he can build up his reputation at the very first stage, where the true state is \( u \), by telling the truth. But if the expert’s initial reputation \( \mu \) is low, then the message \( n \) in the first stage is very likely to be the opportunist expert’s deceit, and thus the receivers may prefer to reject the treatment even though they observe the message \( n \). Thus, the expert should be telling the truth with a sufficiently high probability if he wants to receive positive stage game payoffs while building up his reputation. More formally, the receiver’s expected payoff of approving the treatment conditional on the event that she observes the message \( n \) in the first stage is

\[
EU_r(a|n) = -v_u \frac{\pi(1 - \mu)(1 - \sigma_e)}{\pi(1 - \mu)(1 - \sigma_e) + (1 - \pi)} + v_n \frac{(1 - \pi)}{\pi(1 - \mu)(1 - \sigma_e) + (1 - \pi)}.
\]

whereas the receiver’s expected payoff of rejecting the treatment when she observes the message \( n \) is simply \( EU_r(r|n) = 0 \). Hence, the receiver prefers to approve the treatment if \( EU_r(a|n) \geq 0 \), or equivalently

\[
\sigma_e \geq 1 - \frac{v_n(1 - \pi)}{v_u \pi(1 - \mu)} = \frac{\mu^* - \mu}{1 - \mu}
\]  

(3)

holds. Thus, inequality (3) guarantees that approving the treatment is an optimal for a receiver when she observes the message \( n \).

For some values of the primitives, in particular when \((\mu^*)^2 \leq \mu \) holds, the inequalities (2) and (3) can hold simultaneously. In this case, the expert’s best equilibrium dictates that he must tell the truth when the state is \( u \) only once with probability \( \sigma_e = \frac{\mu^* - \mu}{1 - \mu} \). However, if \((\mu^*)^2 > \mu \), then the inequalities (2) and (3) do not hold simultaneously. In this case, the expert can (and should) build up his reputation gradually (in more than one stage).

6In the proof of Proposition 3, I show that building up his reputation gradually is what the expert should be doing in his best equilibrium if \( 0 < \mu < \mu^* \).

Therefore, to calculate the shortest time that is required for the expert to build her reputation gradually up to the critical level \( \mu^* \), let \( \mu_0 = \mu \) and for all \( t \geq 0 \) define \( \sigma_e^t = \frac{\mu^* - \mu}{1 - \mu} \) and \( \mu_{t+1} = \frac{\mu}{\mu + (1-\mu)\sigma_e^t} \) recursively. The term \( \sigma_e^t \) represents the probability that
the expert tells the truth conditional on observing the state $u$ for the $(t+1)^{th}$ time, and $\mu_t$ represents the expert’s updated reputation given that the receiver observe the true state $u$ for the $t^{th}$ time.

If the expert observes the state $u$ for $k$ times and tells the truth at all times according to $\sigma_t$’s as given above, then his reputation reaches $\mu_k = \frac{\mu}{\mu+(1-\mu)\prod_{t=1}^{k-1}\sigma_t}$. The expert will stop building up her reputation whenever $\mu_k \geq \mu^*$ holds, which is equivalent to $\prod_{t=1}^{k-1}\sigma_t \leq \frac{(1-\mu^*)\mu}{\mu^*(1-\mu)}$. Hence, the shortest time required for the expert to build up her reputation to $\mu^*$—while the receivers prefer to approve the treatment conditional on observing the message $n$—is defined by

$$K^* = \min \left\{ k \in \mathbb{Z}^+ \mid \prod_{t=0}^{k-1}\sigma_t \leq \frac{(1-\mu^*)\mu}{\mu^*(1-\mu)} \right\}. \quad (4)$$

By using this definition of $K^*$, it is rather easier to show that $K^*$ is the smallest of the natural numbers $k$, satisfying $(\mu^*)^{k+1} \leq \mu$, and I show this last step in the proof of Lemma 1.

Note that $K^*$ increases with $\mu^*$ but decreases with $\mu$. Therefore, if the expert’s initial reputation (i.e., $\mu$) is higher, then he needs less time to build up his reputation. If the level of trust the expert needs to possess in order to deceive the receivers (i.e., $\mu^*$) is higher, then the expert needs more time to build up his reputation. Recall that $\mu^*$ is positively correlated with $\nu_u$ and $\pi$, but negatively related with $\nu_n$. Therefore, if the expected return of the treatment is higher (i.e., closer to 0), then $\mu^*$ is lower, and thus, the expert needs less time to build up his reputation.

**Proposition 3.** Suppose that $\beta \in (0,1]$ and $\mu > 0$. For sufficiently high values of $\delta$, deception is consistent with equilibrium, and the expert’s best equilibrium payoff is

$$V_e = (1 - \alpha_\beta) v^f_e + \alpha_\beta v^d_e$$

where $v^d_e \equiv v_e \left(\frac{1+\frac{\pi\delta(1-\pi)}{1-(1-\pi)\delta}}{1-(1-\pi)\delta}\right)^{NG}$ and $\alpha_\beta \equiv \left[\frac{\pi\delta}{1-(1-\pi)\delta}\right]^{NG}$.

Together with Proposition 2, the last result shows that deception is consistent with equilibrium for all values of $\mu \in [0,1)$ and $\beta \in (0,1]$. However, deception would benefit the expert seller only if $\mu > 0$. For all values of $\beta \in (0,1]$ and $\mu > 0$, the term $\alpha_\beta$ is in $(0,1]$, and so, the expert’s best equilibrium payoff is a convex combination of two numbers; $v^f_e$ and $v^d_e$. The term $v^f_e$ is the expert’s payoff in the fully revealing equilibrium. We know from Proposition 2 that $v^f_e$ is the expert’s best equilibrium payoff when $\mu = 0.$
On the other hand, the term $v_e^d$ is the expert’s best equilibrium payoff when the expert’s initial reputation $\mu$ is higher than the threshold level $\mu^*$: when $\mu \geq \mu^*$, the expert does not need to build his reputation to deceive the receivers (i.e., $N_G$ is 0), and thus, $\alpha \beta$ is 1 and $V_e$ is equal to $v_e^d$. For higher values of $N_G$ (i.e., the expert requires longer times to build his reputation), the parameter $\alpha \beta$ takes smaller values. Thus, the expert’s best equilibrium payoff gets closer to his payoff in the fully revealing equilibrium $v_e^f$.

The expert’s best equilibrium strategies have three possible layers. The expert and the receivers start the game in the reputation building phase: the expert sends the message $n$ with certainty if the true state is $n$ and sends the message $u$ with a positive probability that is less than 1 if the true state is $u$. The players move to the deception phase whenever the expert’s reputation exceeds the threshold $\mu^*$ (which happens in $N_G$ observed stages). In this phase, the expert sends the message $n$ regardless of the true state. The deception phase ends whenever the receivers observe the expert’s deceit, after which the players move to the truthful reporting phase. In this phase, the expert and the receivers play their fully-revealing equilibrium strategies.

**Corollary 1.** The function $V_e$, indicating the expert’s best equilibrium payoff, is maximized when

$$\beta = \frac{N_G(1 - \delta)}{\delta \pi}.$$

First note that $\beta$ decreases with $\delta$. That is, a more patient expert prefers weaker monitoring. Second, higher $\mu$ reduces $N_G$, and thus decreases $\beta$. Therefore, the expert with a higher initial reputation reaches the threshold level of reputation faster and prefers a weaker monitoring technology. Equivalently, the expert with a low initial reputation prefers stronger monitoring system to reach the threshold level of reputation earlier. Third, if $\pi$ increases, then $\beta$ decreases. Therefore, if the likelihood that the treatment is unnecessary is higher, then the expert prefers a weaker monitoring technology because he wants to reduce the likelihood of getting caught. Finally, if the receiver’s expected return from the treatment increases (i.e., $\mu^*$ is lower, and thus $N_G$ is lower), then the expert prefers a weaker monitoring technology.

In fact, if the expert’s initial reputation is high enough, in particular $\mu^* \leq \mu$, then no monitoring (i.e., $\beta = 0$) would yield the highest payoff to the expert. However, when the expert needs to build up his reputation to deceive the receivers (i.e., $\mu < \mu^*$), then no monitoring is not in his best interest. The expert prefers to be monitored perfectly while he builds up his reputation for honesty. Once he reaches the threshold level of reputation, the expert prefers not to be monitored by the receivers so that he can deceive
the receivers forever. Overall, there is an inverted U-shaped relationship between the expert’s best equilibrium payoff and the strength of the monitoring technology (i.e., \( \beta \)).

**Corollary 2.** In the expert’s best equilibrium, the expected number of stages that the expert should be truthful to build his reputation up to \( \mu^* \) is \( \frac{N_G}{\delta^*} \) given that \( 0 < \mu < \mu^* \). Furthermore, expected number of stages that the expert deceives the receivers is \( 1/\beta \).

The shortest (expected) amount of time required for the expert to build up his reputation decreases with the expert’s initial reputation \( \mu \), with the monitoring strength \( \beta \), and with the receiver’s expected return from the treatment. On the other hand, the expected length of deception depends only on the monitoring technology. Consistent with the intuition, it decreases with the monitoring technology. However, the relationship has degree of \(-1\). The length of deception is short especially when the monitoring is strong (i.e., \( \beta \) is farther from \( 0 \)). In the next section, I will show that the length of deception would be significantly longer when both the receiver and the expert are long-lived agents.

4. **A Benchmark Result with a Long-Lived Receiver**

In this section, I provide a benchmark result, not a complete analysis, for the case where the receiver is also a long-lived agent, and show that the expert could deceive the receiver for an unlimited period. For this purpose, I suppose that both the expert and the receiver are long-lived agents with the common discount factor \( \delta < 1 \). I restrict my attention to the case where the players can perfectly monitor the history of the game and the expert is known to be the opportunist player. That is, the players’ actions and the true state at all stages are observable by the players and \( \mu = 0 \). The reason for this restriction is the intuition that the expert can deceive the receiver for a longer period of time when the monitoring is not perfect or when the expert has reputation for honesty.

The next result proves that the expert can deceive the receiver for an unlimited period even though he is known to be the opportunist type, and his deception is observable by the receiver. Thus, compared with the results in the previous section, we conclude that the short-lived receivers’ incentives protect them against recurrent deceptions as long as their monitoring technology is transparent enough.

**Proposition 4.** For sufficiently large values of \( \delta < 1 \), there exists an equilibrium of the repeated sender-receiver game in which the expert deceives the long-lived receiver during an unbounded length of time.

\(^7\)By deception I mean full, not partial, deception.
As is standard in repeated games, equilibrium with indefinite period of deception is a result of mutual trust. The expert deceives the receiver for one period, for instance, and then reports truthfully for a while. As long as the expert balances the ratio of deception and truthful reporting in a way that the receiver’s continuation payoff is never negative, the receiver never punishes the expert’s deceptions. Thus, the circle of deception and truthful reporting, which relies on mutual trust, could be repeated indefinitely. However, such a circle of mutual trust between the expert and the short-lived receivers does not yield the expert a payoff that is higher than $v^f$. This is true because the short-lived receivers do not care about any future “reward” the expert may offer to their successors, and thus, short-lived receivers will never approve the treatment unless there is sufficient level of truth in the expert’s advice.

5. Concluding Remarks and Related Literature

Asymmetric information is an important part of the experience goods and credence goods markets. The informational problem between the sellers and the customers may give rise to inefficiencies, such as under and overtreatment, or overpricing. In the long run, such mistreatments also lead to market breakdown if customers, for example, postpone car repairs or medical checkups because of the poor services they have received or high prices they have paid in the past.

The model adopted in this study may also be applied to various other situations. Politicians, for example, share the main characteristics of the experience and credence goods because quality (whether a politician delivers what he promises to voters) can only be evaluated through experience. Likewise, many search goods have became experience good because of e-trade websites. Online customers are not always sure what is coming out of box, if it will be delivered on time or if return will occur with hassle.

The main message of the results of this study is that deception during an unbounded length of time in a relationship between a long-lived expert seller and his customers is a zero-probability event if the customers are short-lived agents. However, it is a probability one event (in some of the equilibrium) when the customers are long-lived agents.

Semantically a “trust-based” relationship can have no demand on the righteousness or honesty of the actors. Trust is one party’s ability to accurately predict the actions of another party. Thus, mutual trust between a seller and his customer is a situation where each conforms with his/her opponent’s expectations. In that regard, a “bad” equilibrium is also a situation of mutual trust. Honesty builds trust, and honesty-based trust may sustain a long-run relationship. However, there are many other factors that would sustain
a long-run relationship. Customers may be loyal to a seller if switching costs are high, if there are few satisfactory alternatives, and if there are bonds keeping them in the relationship. The existence of these bonds acts as an exit barrier. There are several types of bonds, such as legal (contracts), technological (shared technology), economic (dependence, loyalty premiums), geographical, social, or cultural bonds. Potentially, a customer’s expectations in a relationship would be less ambitious when she is constrained with such an exit barrier, and thus, disregarding a deception in return for future rewards would be consistent with a trust-based long-run relationship.

Trust is a forward-looking concept because it is one person’s ability to accurately predict another’s behavior. Thus, the relationship between a long-lived sender and a long-lived receiver represents a trust-based relationship. On the other hand, reputation is not a prediction of future, but knowledge of the past. Reputation is a memory tied to a specific identity. It is a collectively agreed-upon version of how history has taken place. Therefore, the relationship between the sender and the short-lived receivers represents a reputation-based relationship. In this regard, another interpretation of the results is that trust-based relationships would be more vulnerable to fraud than reputation-based relationships.

A seminal paper by Darby and Karni (1973) investigates how market conditions affect the equilibrium amount of mistreatment in credence goods markets. Wolinsky (1993) demonstrates how cheating can be eliminated when customers search for second opinions or experts have reputation concerns. Emons (1997, 2001) study how the price mechanism can discipline experts to practice honestly. Pesendorfer and Wolinsky (2003) study whether a competitive sampling of opinions makes it attractive for experts to provide costly but unobservable diagnostic effort. Alger and Salanie (2006) study under which conditions sellers defraud customers to keep them uninformed, as this deters them from seeking a better price elsewhere. Fong (2005) studies which customers the expert sellers defraud if the customers have heterogeneous and identifiable characteristics (e.g., valuations for treatments or costs of treatment). Dulleck and Kerschbamer (2006) provide an excellent survey for the literature on credence goods.

These theoretical questions are not discussed in this study, as I abstain from competition, search costs, the seller’s diagnosis or pricing efforts, or consumer heterogeneity to keep the model as succinct as possible. The current model differs from the literature on credence goods in two important aspects. First, I model the interaction between a seller and his customer(s) as a simple sender-receiver (or a cheap talk) game, where the seller’s payoff does not directly depend on his messages. This modeling choice eliminates all the complications that would arise when we include, for example, the expert seller’s
pricing decision. Second, I consider a repeated interaction between the seller and his customer(s), which provides a fruitful platform to study the expert seller’s long-run and short-run trade-offs of mistreating his customers.

Following the seminal study by Crawford and Sobel (1982), sender-receiver (or cheap talk) games have became a natural framework to study issues of information transmission between an informed sender (expert) and an uninformed decision maker (receiver). Sobel (2011) provides a very detailed survey of this literature. Unlike the usual treatment in the sender-receiver games literature, I consider an infinite-horizon repeated game. Aumann and Hart (2003) and Krishna and Morgan (2004), for example, consider dynamic sender-receiver games. However, only the talk is repeated in their settings. Golosov, Skreta, Tsyvinsky, and Wilson (2009) study strategic information transmission game in a finite-horizon, dynamic Crawford and Sobel setup. The main result of their study is that fully-revealing equilibrium exists when both the expert and the receiver are long-lived and fully patient players.

Sobel (1985) considers a reputational sender-receiver game where the talk between the expert and the receiver is repeated finitely many stages, and both players are fully patient and long lived. His study assumes that the receiver is uncertain about the bias of the expert—she is either the “friendly” type, whose preferences are perfectly aligned with the receiver, or the “enemy” type, who has completely opposed preferences to the receiver. The main result of his study is that deception is sustainable in equilibrium only if the expert has sufficiently high reputation of being the “friendly” type, and deception would occur only once. Ottaviani and Sorensen (2006, 2006b) and Morris (2001) also investigate reputational sender-receiver games where the expert’s bias is unknown to the receiver. The main message of these studies is that truth telling is incompatible with equilibrium when the expert is sufficiently concerned about her reputation.

Benabou and Laroque (1992) study an infinitely repeated sender-receiver game between an expert and multiple audiences (i.e., the public). However, the players in their model have significantly different incentives. The expert receives an informative signal about the true state of the world. Unlike the current model, the expert in Benabou and Laroque (1992) plays a trading game with her audiences right after sending her public message and directly affects her own payoff. In particular, the expert is an insider who manipulates her audiences’ opinion through her cheap talk messages and trades with them in a purely speculative market. Benabou and Laroque (1992) show that the expert will deceive her audiences (who are short-lived agents) during an unbounded length of time. They conclude, contrary to the results of the present study, that an expert with very low reputation for honesty will make no significant attempt to build her reputation, and the
issue of whether intermediate reputations are worth improving by “investing in truth” remains unresolved.

Appendix

Proof of Proposition 1. To show that there does not exist a fully-revealing equilibrium, suppose for a contradiction that there exists one. Because the expert reports truthfully, $P(m|m) = 1$ for any $m \in S$, and thus, it is optimal for the receiver to play $a$ (or $r$) when the expert sends the message $n$ ($u$). Therefore, the expert’s equilibrium payoffs would be $0$ and $v_e$ when the true states are $u$ and $n$, respectively. However, the opportunist expert would prefer to deviate and send the message $n$ whenever the true state is $u$, contradicting with the optimality of the equilibrium.

Next, I will show that for all values of $\mu$, an influential equilibrium strategy profile must have the following form: the opportunist expert sends the message $n$ irrespective of the true state and the receiver approves (and rejects) the treatment when she observes the message $n$ (respectively $u$). This strategy profile is clearly deceitful. I will also show that such a strategy profile forms a Perfect Bayesian Equilibrium (PBE) if and only if $\mu \geq \mu^*$. 

Suppose that there exists a PBE strategy profile $\sigma$ in which after some message realization the receiver plays $a$ with positive probability. Given the expert’s strategies $\sigma_e(u)$, $\sigma_e(n)$, the receiver’s best response correspondences are calculated as follows: Suppose first that the receiver observes message $u$. Then, the receiver’s expected payoff of playing $a$ and $r$ are given by

$$EU_r(a|u) = -v_u\left(\frac{\pi\mu + \pi(1-\mu)\sigma_e(u)}{\pi\mu + \pi(1-\mu)\sigma_e(u) + (1-\pi)(1-\mu)\sigma_e(n)}\right) + v_n\left(\frac{(1-\pi)(1-\mu)\sigma_e(n)}{\pi\mu + \pi(1-\mu)\sigma_e(u) + (1-\pi)(1-\mu)\sigma_e(n)}\right)$$

$$EU_r(r|u) = 0.$$

Therefore,

$$BR_r(\sigma_e|u) = \begin{cases} 1, & \text{if } \sigma_e(u) < \frac{v_n(1-\pi)(1-\mu)\sigma_e(n) - v_u \pi \mu}{v_u \pi (1-\mu)} \\ [0,1], & \text{if } \sigma_e(u) = A \\ 0, & \text{otherwise.} \end{cases}$$

That is, the receiver plays $a$ with certainty whenever $\sigma_e(u) < A$ and $r$ with certainty if
\[ \sigma_e(u) > A. \] Now, suppose that the receiver observes the message \( n \). Then,

\[
EU_r(a|n) = -v_u \left[ \frac{\pi(1-\mu)(1-\sigma_e(u))}{\pi(1-\mu)(1-\sigma_e(u)) + (1-\pi)(1-\mu)(1-\sigma_e(n))} \right]
+ v_n \left[ \frac{(1-\pi)\mu}{\pi(1-\mu)(1-\sigma_e(u)) + (1-\pi)(1-\mu)(1-\sigma_e(n))} \right],

\[
EU_r(r|n) = 0.
\]

Therefore,

\[
BR_r(\sigma_e|n) = \begin{cases} 
1, & \text{if } \sigma_e(u) > \frac{v_u\pi(1-\mu) - v_n(1-\pi)\mu - v_n(1-\pi)(1-\mu)(1-\sigma_e(n))}{v_u\pi(1-\mu)} \\
[0,1], & \text{if } \sigma_e(u) = B \\
0, & \text{otherwise.}
\end{cases}
\]

That is, the receiver plays \( a \) with certainty whenever \( \sigma_e(u) > B \) and \( r \) if \( \sigma_e(u) < B \). Note that we have \( A < B \) because the inequality (1) holds. Recall that \( \sigma \) is such that the receiver plays \( a \) after some message realization with positive probability. There are five exhaustive cases regarding the value of \( \sigma_e(u) \) relative to \( A \) and \( B \):

(i) Assume that \( \sigma_e(u) < A < B \). Then the receiver plays \( a \) and \( r \) when she receives the messages \( u \) and \( n \), respectively. Therefore, the expert’s payoffs of sending messages \( u \) and \( n \) are \( v_e \) and 0, respectively. Then, the optimality of equilibrium implies that the expert will send message \( u \) regardless of the true state, that is \( \sigma_e(u) = 1 \) and \( \sigma_e(n) = 1 \). However, we have \( B = 1 - \frac{v_u(1-\pi)}{v_u\pi(1-\mu)} \) when \( \sigma_e(n) = 1 \), contradicting the initial assumption \( \sigma_e(u) < B \).

(ii) Assume that \( A = \sigma_e(u) < B \). Then the receiver is indifferent between \( a \) and \( r \) when the expert sends the message \( u \). However, the receiver chooses \( r \) when the expert sends the message \( n \). Because the receiver plays \( a \) with positive probability in \( \sigma \), we must have that the receiver plays \( a \) with positive probability after observing message \( u \). Therefore, the expert’s payoffs of sending messages \( u \) is positive whereas his payoff of sending message \( n \) is 0. Then, once again, the optimality of equilibrium implies that the expert will send message \( u \) regardless of the true state, that is \( \sigma_e(u) = 1 \) and \( \sigma_e(n) = 1 \). Similar to the previous case, we reach a contradiction because \( \sigma_e(u) = 1 < B \) and \( B \) is strictly less than 1 for \( \sigma_e(n) = 1 \).

(iii) Assume that \( A < \sigma_e(u) < B \). Then the receiver plays \( r \) regardless of the expert’s message, contradicting that the receiver plays \( a \) with positive probability in \( \sigma \).

(iv) Assume that \( A < \sigma_e(u) = B \). Then the receiver plays \( r \) when the expert sends the message \( u \), and she is indifferent between \( a \) and \( r \) when the expert sends the message
Because the receiver plays $a$ with positive probability in $\sigma$, we must have that
the receiver plays $a$ with positive probability after observing message $n$. Therefore,
the expert’s payoffs of sending messages $n$ is positive whereas his payoff of sending
message $u$ is 0. Then, the optimality of equilibrium implies that the expert will send
message $n$ regardless of the true state, that is $\sigma_e(u) = 0$ and $\sigma_e(n) = 0$. For these
values of $\sigma_e(u)$ and $\sigma_e(n)$, we have $A < 0$ and $B = 1 - \frac{v_n(1-\pi)}{v_u \pi (1-\mu)} = 0$ if and only if
$\mu = \mu^*$.

(v) Assume that $A < B < \sigma_e(u)$. Then the receiver plays $r$ when the expert sends the
message $u$ and $a$ otherwise. Therefore, the expert’s payoffs of sending messages $n$
and $u$ are $v_e$ and 0, respectively. Then, the optimality of equilibrium implies that the
expert will send message $n$ regardless of the true state, that is $\sigma_e(u) = 0$ and $\sigma_e(n) = 0$. For these
values of $\sigma_e(u)$ and $\sigma_e(n)$, we have $A < 0$ and $B = 1 - \frac{v_n(1-\pi)}{v_u \pi (1-\mu)} < 0$ if
and only if $\mu > \mu^*$.

The first three cases show that we cannot have an influential equilibrium where the
expert sends message $u$. The last two cases show that in an influential equilibrium,
the expert sends message $n$ with certainty regardless of the true state and the expert’s
reputation must be no less than $\mu^*$.

**Proof of Proposition 2.** Consider the following strategy profile. There are two phases;
the coordination and the punishment. In the coordination phase, the opportunist expert
truthfully reports the true state and the receivers approve the treatment when they observe
the message $n$ and reject it if they observe $u$. In the punishment phase, the expert always
sends message $n$ and the receivers reject the treatment regardless of the message they
observe. The repeated game starts with the coordination phase and the players stay in
this phase unless the expert deviates. Once the expert deviates and the receivers observe
this deviation, then the game moves to the punishment phase and stays there for the rest
of the game.

Next, I will discuss that this strategy profile is a PBE of the repeated sender-receiver
game. If the game moves to the punishment phase, then it should be public knowledge
that the expert is the opportunist type with certainty. The punishment phase strategies
forms a babbling equilibrium and yield 0 payoff for both players. By Proposition 1,
because the expert’s reputation is lower than $\mu^*$, a short-lived receiver will never approve
the treatment. Moreover, if the expert deviates in the punishment phase, then he cannot
improve his payoff either. Therefore, the punishment phase strategies are optimal for
both players. Note that this is the worst payoff the expert would make in the repeated
game.

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In the coordination phase, the expert’s continuation payoff is
\[(1 - \pi)v_e\delta = 0 + (0\pi + (1 - \pi)v_e)\delta + (0\pi + (1 - \pi)v_e)\delta^2 + \ldots\]
if the expert observes state \(u\), and is \(v_e + \frac{(1-\pi)v_e\delta}{1-\delta}\) if the expert observes state \(n\). Therefore, the expert’s payoff in the coordination phase is
\[\left[(1 - \pi)v_e + \frac{(1 - \pi)v_e\delta}{1 - \delta}\right] = \frac{(1 - \pi)v_e}{1 - \delta} \equiv v_e^f.\]
The expert has no incentive to deviate when the state is \(n\). However, if he deviates when the state is \(u\), then his deviation will lead to the continuation payoff of \(v_e + (1 - \beta)v_e^f\). Therefore, the expert does not deviate from his coordination phase strategies if and only if \(\frac{(1-\pi)v_n\delta}{1-\delta} \geq v_e + (1 - \beta)v_e^f\), or equivalently \(\delta \geq \delta_\beta \equiv \frac{1+(1-\beta)(1-\pi)}{1+(1-\pi)\beta+(1-\beta)(1-\pi)}\).

Finally, given the expert’s strategy, a short-lived receiver’s expected payoff of following her strategy is \(0\) when the true state is \(u\) and \(v_n\) when the true state is \(n\). However, if she deviates, her payoff will be \(-v_u\) when the true state is \(u\) and \(0\) when the true state is \(n\). Thus, coordination phase strategies are also optimal for each short-lived receiver.

Next, I will show that deception does not increase the expert’s payoff. Let \(h^t\) be a history (possibly the null history) in which the expert is known to be the opportunist type. First, I will show that there is no equilibrium of the continuation game following the history \(h^t\) in which the receiver approves the treatment with positive probabilities after observing both messages. Suppose for a contradiction that there exists an equilibrium in which \(\sigma_e(h^t,m)(a) \in (0,1)\) for each \(m \in M\). Recall the receiver’s best response correspondences from the proof of Proposition 1. Conditional on observing the message \(u\), the receiver approves the treatment if \(\sigma_e(u) \leq \sigma_e(n)\frac{v_u(1-\pi)}{v_u(1-\pi) + v_n(1-\pi)} : = A\). Note that the strategy \(\sigma_e(m)\) in the one stage game corresponds to \(\sigma_e(h^t,m)(u)\) in the repeated sender-receiver game. However, when the receiver observes message \(n\), she approves the treatment if \(\sigma_e(u) \geq 1 - (1 - \sigma_e(n))\frac{v_u(1-\pi)}{v_u(1-\pi) + v_n(1-\pi)} : = B\). Therefore, the receiver approves the treatment with a positive probability regardless of the message she observes if and only if \(B \leq \sigma_e(u) \leq A\) holds. However, since the inequality \([1]\) holds, we have \(A < B\) for all values of \(\sigma_e(n)\), that yields the desired contradiction.

Therefore, if the expert wants to make receiver approve the treatment even when the true state is \(u\), he must send the message \(n\) regardless of the true state. However, full deception is not consistent with equilibrium. If the receiver believes that the expert will deceive her with certainty at some stage after \(h^t\), then the short-lived receiver prefers to reject the treatment at that stage. However, partial deception would be consistent with
equilibrium. In particular, we know from the receiver’s best response correspondences that if the expert sends the message \( n \) with certainty when the true state is \( n \) and with a probability \( \sigma_e(h, u)(n) \leq \frac{v_n(1-\pi)}{v_u\pi} \) when the true state is \( u \), then the receiver prefers to approve the treatment if she observes the message \( n \). Next, I will show that partial deception does not improve the expert’s payoff.

Let \( V \) be the highest continuation payoff of the expert in any equilibrium following a history \( h^t \) in which the expert is known to be the opportunist type. Then, the expert’s expected payoff if the true state is \( n \) in stage \( t + 1 \) is at most \( v_e + \delta V \). However, if the true state is \( u \) in stage \( t + 1 \), then the expert’s continuation payoff of telling the truth is no more than \( 0 + \delta V \). In equilibrium where the expert tells the truth with a positive probability when the true state is \( u \), the expert’s continuation payoff of lying and telling the truth must be the same. Hence, her continuation payoff when the state is \( u \) should be no more than \( \delta V \). Thus, \( V \) must be less than or equal to \( (1-\pi)(v_e + \delta V) + \pi \delta V \), implying that \( V \leq \frac{(1-\pi)v_e}{1-\delta} \). Hence, \( v_e^t \) is the upper boundary for the expert’s equilibrium payoffs following the history \( h^t \).

Finally, I would like to argue (rather informally) that partial deception is consistent with equilibrium. That is, there exists equilibrium with partial deception. I will support my claim for \( \beta = 1 \) and for a carefully selected \( \delta \). There is no reason to doubt that more detailed strategies will exist for other values of \( \beta \in (0, 1) \) and \( \delta \) that is high enough. Consider the following strategy profile. The expert sends message \( n \) as long as he observes \( n \). When the expert observes \( u \) for the first time, he sends message \( n \) with probability \( \sigma_e(h, u)(n) \leq \frac{v_n(1-\pi)}{v_u\pi} \) and sends message \( u \) with the remaining probability. The receiver approves the treatment as long as she observes message \( n \). However, when the expert is get caught lying, then both players move to the punishment phase where they play their babbling equilibrium strategies forever. However, when the expert sends message \( u \) in stage \( t \) for the first time, the expert and the receivers play the fully revealing equilibrium until the receiver approves the treatment \( M \) times after stage \( t \). Once the number of approvals hits \( M \), the players play their babbling equilibrium strategies for the rest of the game. The game moves to the punishment phase in any deviation of the expert. It is rather easy to verify that such a strategy profile with partial deception forms an equilibrium when \( M \) and \( \delta \) are selected carefully.

**Proof of Lemma 1.** Suppose that \( \mu \in (0, \mu^*) \). We know that \( \mu^* = 1 - \frac{v_n(1-\pi)}{v_u\pi} \), and \( \sigma^t_e = 1 - \frac{v_n(1-\pi)}{v_u\pi(1-\mu_t)} \). Therefore, we can write

\[
\sigma^t_e = \frac{\mu^* - \mu_t}{1 - \mu_t}.
\]
Moreover, we know that $\mu_{t+1} = \frac{\mu_t}{\mu_t + (1 - \mu_t) \sigma_t}$ with $\mu_0 = \mu$, and thus, the recursive structure implies that

$$\mu_t = \frac{\mu}{\mu + (1 - \mu)(\sigma_0^t \sigma_1^t \cdots \sigma_{t-1}^t)}.$$  

(6)

The equations in (5) and (6) imply that

$$\sigma_t e = \mu - \mu (1 - \mu) (1 - \mu) (\sigma_0 e \sigma_1 e \cdots \sigma_{t-1} e).$$

(7)

Thus, given the starting point $\sigma_0 e = \frac{\mu^* - \mu}{1 - \mu}$ and the equation (7) we can recursively calculate $\sigma_0^t e \sigma_1^t e \cdots \sigma_{k-1}^t$ for any $k \geq 1$ as follows: First $\sigma_0^t e = \frac{(\mu^*)^3 - \mu}{1 - \mu}$. Using the last equation and (7) we find that $\sigma_0^2 e \sigma_1^2 e = \frac{(\mu^*)^2 - \mu}{1 - \mu}$. Repeating this process yields

$$\Pi_{t=0}^{n-1} \sigma_t e = \frac{(\mu^*)^k - \mu}{1 - \mu}.$$  

By using the definition of $K^*$ in (4), it is rather easier to show that it is the smallest of the natural numbers $k$ satisfying $(\mu^*)^{k+1} \leq \mu$, which is equivalent to $k \geq \frac{\ln \mu}{\ln \mu^*} - 1$.

**Proof of Proposition 3.** In what follows, I will describe some strategies and show that they are equilibrium strategies of the repeated game $G^\infty$ that yield the expert the highest possible payoff. While I describe some (parts) of these strategies, I use a public randomization device simply because the description of these strategies are much shorter and easier with it. Fudenberg and Maskin (1991) show that one can actually get rid of the public randomization device for sufficiently high $\delta$, by appropriate choice of which periods to play each action profile involved. For the same reason, we can also get rid of the public randomization device and prove the next result. I will start with presenting two lemmas that will be helpful for the proof of Proposition 3.

**Lemma 2.** Consider a history $h^t$ of the repeated sender-receiver game $G^\infty$ in which the expert is known to be the opportunist type. For any payoff $v$ in the range $[0, v^f]$ and sufficiently large values of $\delta < 1$, there exists an equilibrium of the continuation game following the history $h^t$ in which the expert’s payoff is $v$.

*Proof.* We know from Proposition 2 that a fully-revealing and influential equilibrium exists for the continuation game following the history $h^t$, where the expert’s payoff is $v^f$. Likewise, a babbling equilibrium also exists for this subgame, where the receivers reject the treatment regardless of the message and the expert’s payoff is 0. Let $v = \tau v^f$ and $\tau \in (0, 1)$. Consider the following strategy profile: At the beginning of each stage, before the expert observes the true state, both the expert and the receivers observe the outcome
of a public randomization device that has two possible outcomes; A and B. Outcome A occurs with probability $\tau$ and outcome B occurs with probability $1 - \tau$. When the outcome is A in stage $t$, both players play their fully-revealing equilibrium strategies in this stage, where the expert tells the truth and the receiver approves (or rejects) the treatment when she observes the message $n$ (or $u$). However, when the outcome is B, then players play their babbling equilibrium strategies, where the expert always sends the message $n$ and the receivers always reject the treatment. If one of the players deviates and if the receivers publicly observe this deviation, then both players move to the punishment phase, where the expert and the receiver play their babbling equilibrium strategies forever.

According to this strategy profile, the expert’s game payoff is $\tau v_e (1 - \pi) v_e 1 - \delta$, which is equal to $\tau v_e$ as required. Similar to the arguments in the proof of Proposition 2, the punishment phase strategies form a babbling equilibrium. Because the expert is known to be the opportunist type, the receiver has no incentive to approve the treatment and the expert has no incentive to tell the truth when the public randomization outcome is B.

However, if the outcome is A, then the expert has no incentive to deviate when the true state is $n$. If he deviates in state $u$ and sends the message $n$, then his continuation payoff will be $v_e + \frac{\delta \tau (1 - \beta)(1 - \pi) v_e}{1 - \delta}$. If he tells the truth, his continuation payoff is $\frac{\delta \tau (1 - \pi) v_e}{1 - \delta}$. Therefore, the expert does not deviate from his strategy if and only if $\frac{\delta \tau (1 - \pi) v_e}{1 - \delta} \geq v_e + \frac{\delta (1 - \beta)(1 - \pi) v_e}{1 - \delta}$, or equivalently $\delta \geq \delta^* \equiv \frac{1}{1 + \pi (1 - \beta)} \in (0, 1)$. This completes the proof.

**Lemma 3.** Suppose that $\mu \geq \mu^*$ and $\beta \in (0, 1]$. The expert’s best equilibrium payoff in the repeated sender-receiver game $G^\infty$ is $v_e^d = v_e \left( \frac{1 + \pi \delta (1 - \pi)}{1 - (1 - \pi \beta) \delta} \right)$.

**Proof.** First, I will show that the payoff $v_e^d$ can be supported in equilibrium. Then I will argue that it is the highest expected payoff that the expert can achieve in any PBE of the repeated sender-receiver game. There are three phases of the strategy profile; deception, truthful-reporting and punishment. Players start in the deception phase, where the expert sends the message $n$ regardless of the true state until he gets caught lying. Once he gets caught lying by the receivers, the players move to the truthful-reporting phase, where the expert always tells the truth, and stay there forever. In the deception and truthful-reporting phases, the receivers always approve the treatment when they observe the message $n$ and reject the treatment otherwise. If the expert ever deviates in any phase and if his deviation is detected by the receivers, then the players move to the punishment phase and stay there forever, where the receivers always reject the treatment and the expert always sends the message $n$. If a receiver deviates in the deception phase, then
the players will continue to stay in the deception phase. However, if a receiver deviates in any other phase, then the players move to the punishment phase and stay there forever.

The expert’s payoff under this strategy profile is calculated by solving the following recursive equation

\[ V = \pi \left( v_e + \delta \left[ (1 - \beta) V + \beta v_e^f \right] \right) + (1 - \pi) \left( v_e + \delta V \right) \]

The first parenthesis is the expert’s continuation payoff when the true state is \( u \). The expert will deceive the receiver in the first stage. Following the deception, the expert will get caught lying with probability \( \beta \), and so, will be truthful forever. In this case, his continuation payoff will be \( v_e^f \). However, the expert will not get caught lying with probability \( 1 - \beta \), in which case the continuation game is identical with the game itself. The second parenthesis is the expert’s expected payoff when the true state is \( n \). The expert does not deceive the receiver in the first stage, and so, the continuation game will be identical to the game itself. The solution of this equation for \( V \) yields the value for \( v_e^d \).

As we already argued before, punishment phase is a babbling equilibrium of the game. In the truthful reporting phase, the receivers know that the expert is opportunist. Thus, as we argued in Proposition 2, the players’ strategies in the truthful reporting phase are optimal as well. Finally, in the deception phase, if the expert deviates and sends the message \( u \), then his payoff will simply be 0. We know from Proposition 1 that each short-lived receiver’s expected payoff of approving the treatment is positive because \( \mu \geq \mu^* \). Therefore, if a receiver deviates in the deception phase and rejects the treatment when she observes \( n \), then she will get 0 payoff. Hence, deception phase strategies are also optimal for both players.

Finally, I will argue that \( v_e^d \) is the highest payoff the expert can achieve in any equilibrium of the repeated sender-receiver game \( G^\infty \). Recall the strategy that is described above. The receiver approves the treatment if the true state is \( n \). Moreover, she also approves the treatment when the true state is \( u \) if the expert’s type has not revealed to the receivers yet. When the expert is get caught lying, the expert’s type will be revealed. Thus, his best equilibrium payoff will be \( v_e^f \) as we proved in Proposition 2. Therefore, a higher game payoff for the expert is possible only if the expert sends the message \( n \) in the deception phase with a probability less than 1 when the true state is \( u \), and thus, reduces the chances that she gets caught lying. This case (i.e., partial deception) would not increase the expert’s payoff as we previously argued. In particular, in case of partial deception, the expert randomizes over the messages in \( M \) at some stage where the true state is \( u \) if and only if he is indifferent between these messages at that stage. When the
true state is \( u \), the expert’s continuation payoff is at most \( v_e + \delta \left[ (1 - \beta)V + \beta \frac{v_e(1-\pi)}{1 - \delta} \right] \) if he lies (i.e., sends the message \( n \)), and thus his continuation payoff must also be at most this much when he tells the truth.

Therefore, if \( V \) is the highest continuation payoff of the expert in any equilibrium following a history where the expert’s reputation is higher than or equal to \( \mu^* \), then

\[
V \leq (1 - \pi)(v_e + \delta V) + \pi \left( v_e + \delta \left[ (1 - \beta)V + \beta \frac{v_e(1-\pi)}{1 - \delta} \right] \right),
\]

implying that

\[
V \leq v_e \left[ \frac{1 + \frac{\pi \beta (1 - \pi)}{V}}{1 - (1 - \pi)\beta} \right].
\]

This completes the proof. 

Now I will start the proof of Proposition 3. First, I will show that the payoff \( V_e \) can be supported in equilibrium. Then I will argue that it is the expert’s highest equilibrium payoff in the game. Consider the following strategy profile \( \sigma \):

(i) The expert always sends the message \( n \) whenever the true state is \( n \).

(ii) Reputation building phase: Let \( h^t \) be a history where (1) the expert’s reputation at the beginning of stage \( t + 1 \) (i.e., \( \mu(h^{t+1}) \)) is strictly positive, but strictly less than \( \mu^* \), and (2) none of the players’ deviation from his/her prescribed strategies has publicly observed by the receivers. In stage \( t + 1 \), following the history \( h^t \), (1) the expert sends message \( u \) with probability \( \sigma_e(h^t, u)(u) = 1 - \frac{v_e(1-\pi)}{v_e\pi(1-\mu(h^t))} \) when the true state is \( u \), and (2) the receivers reject (approve) the treatment if they observe the message \( u \) (\( n \)).

(iii) Let \( h^t \) be a history where (1) none of the players’ deviation from his/her prescribed strategies has publicly observed by the receivers, (2) the true state in stage \( t \) was \( u \), but (3) the expert has lied in stage \( t \) and his true type has revealed to the receivers (i.e., \( \mu(h^t) = 0 \)). In the continuation game following the history \( h^t \), players move to public randomization phase and stay there forever. In this phase, the expert and the receivers play the strategies that are described in the proof of Lemma 2. The public randomization device produce the outcome \( A \) with probability \( \tau_k \equiv \frac{(\delta V(k) - v_e)(1 - \delta)}{\delta \beta (1 - \pi) v_e} \), where

\[
V(k) = v_e^k \left( 1 - \left[ \frac{\pi \beta \delta}{1 - (1 - \pi \beta) \delta} \right]^{N_G - k} \right) + v_d^k \left[ \frac{\pi \beta \delta}{1 - (1 - \pi \beta) \delta} \right]^{N_G - k}
\]

and \( k \leq N_G \) is the number of stages (including stage \( t \)) that the receivers publicly observed in the history \( h^t \), where the true state was \( u \). Thus, in the public randomization phase (1) the expert always tells the truth and the receivers always
approve (reject) the treatment if the message is \( u \) (\( n \)) when the outcome is A, and (2) the expert and the receivers play their babbling equilibrium strategies when the outcome is B.

(iv) Let \( h^t \) be a history where (1) the expert’s reputation \( \mu(h^t) \) is (weakly) higher than \( \mu^* \), and (2) none of the players’ deviation from his/her prescribed strategies has publicly observed by the receivers. In the continuation game following the history \( h^t \), all the players move to deception phase and stay there until the expert gets caught lying. In the deception phase, (1) the expert always sends the message \( n \) (regardless of the true state) and (2) the receivers reject (approve) the treatment if they observe the message \( u \) (\( n \)). Once the expert gets caught lying, all the players move to truthful reporting phase and stay there forever. In the truthful reporting phase, the expert always tells the truth, the receivers reject the treatment if the message is \( u \) and approve it if the message is \( n \).

(v) After any history \( h^t \) where the expert has deviated from his strategies and this deviation is publicly observed by the receivers, players move to the punishment phase and stay there forever. In the punishment phase, the receivers always reject the treatment, and the expert always sends the message \( u \). If a receiver deviates in the deception phase, then the players will continue to stay in the deception phase. However, if a receiver deviates in any other phase, and if the receivers publicly observe her deviation, then the players move to the punishment phase and stay there forever.

First, note that, for any \( \beta \in (0, 1] \), there exists some \( \delta_k < 1 \) such that \( \tau_k \in (0, 1) \) for all \( \delta \geq \delta_k \). \( \tau_k \) is positive because \( v_e < \delta \beta v^d_e < \delta \beta V(k) \) for high values of \( \delta \). Likewise, \( \tau_k < 1 \) because \( V(k) < v^d_e \) and \( \delta \beta v^d_e < v_e + \frac{\delta \beta (1 - \pi) v_e}{1 - \delta} \), implying \( (\delta \beta V(k) - v_e)(1 - \delta) < \delta \beta(1 - \pi)v_e \) and \( \frac{(\delta \beta V(k) - v_e)(1 - \delta)}{\delta \beta(1 - \pi)v_e} < 1 \). Now, I will show that \( \delta \beta v^d_e < v_e + \frac{\delta \beta (1 - \pi)v_e}{1 - \delta} \) holds: Suppose for a contradiction that it does not. That is, \( \delta \beta v^d_e \geq v_e + \frac{\delta \beta (1 - \pi)v_e}{1 - \delta} \). If we insert the value of \( v^d_e = v_e \left( \frac{1 - \delta + \pi \delta \beta(1 - \pi)}{(1 - \delta)(1 - (1 - \pi) \beta)} \right) \) to this inequality and cancel the \( v_e \)'s, then we get

\[
\delta \beta \left( \frac{1 - \delta + \pi \delta \beta(1 - \pi)}{(1 - \delta)(1 - (1 - \pi) \beta)} \right) \geq 1 + \frac{\delta \beta (1 - \pi)}{1 - \delta} \]

. Multiply both sides of this inequality with \( 1 - \delta \) and divide by \( \delta \beta \), and then subtract \( \frac{\delta \pi (1 - \pi)}{1 - (1 - \pi) \beta} \) from both sides to get \( \frac{1 - \delta}{1 - (1 - \pi) \beta} \geq \frac{(1 - \pi)(1 - \delta)}{1 - (1 - \pi) \beta} + \frac{1 - \delta}{\delta \beta} \). Dividing both sides by \( 1 - \delta \) and rearranging the terms yield \( \frac{1}{\delta \beta} \leq \frac{\frac{\pi}{1 - (1 - \pi) \beta}}{1 - (1 - \pi) \beta} \). The last inequality implies \( 1 - \delta + \delta \pi \beta \leq \delta \pi \beta \), which is equivalent to \( 1 \leq \delta \). The last inequality yields the desired contradiction.

Let the term \( V(k) \), where \( 0 \leq k \leq N_G \), represents the expert’s continuation payoff, following a history \( h^t \) where (1) none of the players’ deviation from his/her prescribed strategies has publicly observed by the receivers, and (2) the true state was \( u \) in exactly \( k \)
stages prior to time $t$ (including stage $t$), and these stages were publicly observed by the receivers. Therefore, $V(k)$ is the expert’s continuation payoff when his public reputation is $\mu_k = \frac{\mu_{k-1}}{\mu_{k-1} + (1 - \mu_{k-1})\sigma_k(h^y, u)(u)}$, where $y \leq t$ is the latest publicly observed stage in which the true state was $u$. Thus,

$$V(0) = (1 - \pi)[v_e + \delta V(0)] + \pi\delta[p\beta V(1) + (1 - \beta) V(0)]$$

$$V(1) = (1 - \pi)[v_e + \delta V(1)] + \pi\delta[p\beta V(2) + (1 - \beta) V(1)]$$

$$V(N - 1) = (1 - \pi)[v_e + \delta V(N - 1)] + \pi\delta[p\beta V(N) + (1 - \beta) V(N - 1)]$$

and $V(N) = v_e^d$.

Recall that following the history $h^t$, where the true state was $u$ in exactly $k$ stages that are publicly observed by the receivers, the expert’s reputation is $\mu_k$. The first part of the recursive equation of $V(k)$ (i.e., $[v_e + \delta V(k)]$) is the expert’s continuation payoff conditional on the event that the true state is $u$ in stage $t+1$. In this case, the expert tells the truth in stage $t+1$, and so his stage payoff is $v_e$. The continuation game following stage $t+1$ will be identical to the continuation game following the history $h^t$. Thus, the expert’s reputation is still $\mu_k$, and so, the expert’s continuation payoff following stage $t+1$ is $V(k)$. The second part of the recursive equation $\delta[p\beta V(k+1) + (1 - \beta) V(k)]$ (or $\delta v_e^d$ in case $k = N - 1$) indicates the expert’s continuation payoff if the true state is $u$ in stage $t+1$. The expert tells the truth and his stage game payoff is $0$. With probability $\beta$ the receivers observe stage $t+1$, and so the expert’s reputation will be updated to $\mu_{k+1}$. Therefore, the expert’s continuation payoff will be $V(k+1)$. However, with probability $1 - \beta$ the receivers do not observe stage $t+1$, and thus the expert’s continuation game payoff will be $V(k)$. If $k = N - 1$, then the expert’s reputation will reach a level above $\mu^*$ in stage $t+1$ if this stage is publicly observed by the receivers, and so the expert’s continuation payoff, according to the strategies given in (iv), will be $v_e^d$ (the expert’s highest equilibrium payoff in the deception phase), as we prove in Lemma 3.

The expert’s expected payoff in the repeated sender-receiver game is equal to $V(0)$. In order to find its value, we must solve these $N$ equations recursively. First solve the last equation, which implies $V(N - 1) = \left[1 - \pi\beta\delta \frac{\pi\beta\delta}{1 - (1 - \pi\beta)\delta} \right] v_e^f + \left( \frac{\pi\beta\delta}{1 - (1 - \pi\beta)\delta} \right) v_e^d$. At the end of this process we get

$$V(0) = \left[1 - \left( \frac{\pi\beta\delta}{1 - (1 - \pi\beta)\delta} \right)^{N - 1} \right] v_e^f + \left( \frac{\pi\beta\delta}{1 - (1 - \pi\beta)\delta} \right)^{N - 1} v_e^d,$$

which is equal to $V_e$ as desired.
Next, I will show that the strategy profile $\sigma$ forms a PBE of the repeated sender-receiver game. We already know that the punishment strategies in part $(v)$ and the strategies in $(i)$ are optimal. Lemma 3 shows that the strategies in part $(iv)$ are optimal as well. By Lemma 2, the strategies in part $(iii)$ are optimal because the expert’s continuation payoff after a history $h^t$ that fits to the description in $(iii)$ is $\tau_k \frac{(1-\pi)n_e}{1-\delta}$ (i.e., $\tau_k v^d_e$), and $\tau_k < 1$ as we proved above.

Therefore, all we need to show that the strategies in $(ii)$ are optimal as well. Consider a history $h^t$ where (1) the expert’s reputation at the end of stage $t$ (i.e., $\mu(h^t)$) is strictly positive and strictly lower than $\mu^*$, and (2) none of the players’ deviation has publicly observed by the receivers before. The expert has no incentive to deviate when the true state is $n$ in stage $t + 1$. Therefore, suppose that the true state in stage $t + 1$ is $u$. Furthermore, suppose that $1 \leq k \leq N_G$ is the number of stages (including stage $t$) in the history $h^t$, where the true state was $u$ and the experts publicly observed the previous $k$ stage outcomes. The expert’s continuation payoff if he tells the truth in stage $t + 1$ is $\delta \beta V(k + 1) + \delta (1 - \beta) V(k)$. However, if he lies and sends the message $n$ in stage $t + 1$, then his continuation payoff is $v_e + \delta \beta \tau_{k+1} \frac{(1-\pi)n_e}{1-\delta} + \delta (1 - \beta) V(k)$ because with probability $\beta$ the receivers will catch the expert lying, and thus all will move to public randomization phase starting in stage $t + 2$. Because $\tau_{k+1} = \frac{(\delta \beta V(k + 1) - v_e)/(1 - \delta)}{\delta \beta (1 - \pi)}$, it is easy to check that these two continuation payoffs are the same. That is, the expert is indifferent between lying and telling the truth when the true state is $u$ in stage $t + 1$. Thus, sending the message $u$ with probability $\sigma_e(h^t, u)(u)$ in stage $t + 1$ is a best response for the expert.

Moreover, given the expert’s strategies and the history $h^t$ described in the previous paragraph, a short lived receiver’s expected payoff of approving the treatment conditional on observing the message $n$ is $EU_r(a|n) = -v_n P(u|n) + v_n [1 - P(u|n)]$, where $P(u|n) = \frac{\pi(1-\mu(h^t))|1-\sigma_e(h^t,u)(u)|}{\pi(1-\mu(h^t))|1-\sigma_e(h^t,u)(u)| + (1-\pi)}$. For the value of $\sigma_e(h^t, u)(u)$ that is given above, $EU_r(a|n) = 0$, and thus, the receiver is indifferent between approving and rejecting the treatment whenever she observes the message $n$. Thus, approving the treatment when she observes $n$ and rejecting the treatment when she observes $u$ is a best response strategy for the short-lived receivers. Hence, the strategies in $(ii)$ (together with the strategies in $(iii)$ and $(iv)$) form a PBE of the continuation game.

$V(0) = V_e$ is the highest PBE payoff that the expert can attain in the repeated sender-receiver game $G^\infty$. We can prove this recursively: By Lemma 3, in any equilibrium of the subgame, following a history where the expert’s reputation is higher than $\mu^*$, the expert’s highest payoff is $v^d_e$. Thus, we must have $V_{N_G} = v^d_e$, where $V_{N_G}$ denotes the expert’s highest equilibrium payoff in any continuation game following such a history. If $N_G = 0$, then we are done. If, however, it is positive, then the expert’s highest equilibrium
payoff in any subgame following a history where the expert’s reputation is \( \mu_{NG-1} \), call it \( V_{NG-1} \), must be less than \( (1 - \pi)[v_e + \delta V_{NG-1}] + \pi \delta V_{NG} \). This is true because in the next stage, either the true state will be \( n \), and so the expert’s highest continuation payoff will be \( v_e + \delta V_{NG-1} \), or the true state will be \( u \) and the expert will tell the truth, build his reputation to a level above \( \mu^* \), and receive at most \( 0 + \delta V_{NG} \). Thus, solving all these inequalities recursively will yield that the expert’s highest equilibrium payoff in the game must be less than or equal to \( V_e \).

In the strategy profile that is described to prove Proposition 3, the expert builds his reputation gradually, which delays the payoff \( v_e^d \). However, the expert would play a strategy in which he builds his reputation in few stages (by telling the truth when the true state is \( u \) with a probability less than \( \sigma_e(h^t, u)(u) = \frac{v_u(1-\pi)}{v_u(1-\mu(h^t))} \)). However, if the expert follows a strategy in which he lies with a probability greater than \( \sigma_e(h^t, u)(u) \), then in equilibrium, the receiver certainly rejects the treatment regardless of the message (recall that \( \sigma_e(h^t, u)(u) \) is the probability of truth telling that makes the receivers indifferent between approving and rejecting the treatment). Thus, building reputation faster than \( NG \) stages implies that the expert should give up his positive stage game payoffs until his reputation reaches \( \mu^* \). However, for such a strategy to be a part of an equilibrium strategy, the expert’s continuation payoff of telling the truth and lying (in case the true state is \( u \)) must be the same. Suppose that the expert lies with a very high probability so that he can build up his reputation just in 1 stage by telling the truth when the true state is \( u \). Therefore, the expert’s stage game payoff of sending message \( u \) (when the true state is \( u \)) is 0, and so, his continuation payoff is at most \( 0 + \delta v_e^f \). However, if he lies and gets caught, his continuation payoff will be at most \( 0 + \delta v_e^f \) (by Proposition 2), not \( v_e + \delta v_e^f \). The \( \delta v_e^f \) is lower than \( \delta v_e^d \) for all values of \( \delta \). Thus, if \( 0 < \mu < \mu^* \), then in equilibrium, the expert cannot get a game payoff higher than \( v_e^f \) with a strategy where he does not build his reputation gradually. Equivalently, the expert can attain his highest expected payoff in a strategy profile where he gradually builds his reputation. Since \( NG \) is the shortest time that the expert needs to build up his reputation, \( V_e \) must be the highest payoff the expert can attain in any PBE of the repeated sender-receiver game.

**Proof of Corollary 1.** Proposition 3 indicates that for any \( \beta_0 \in (0, 1) \), there exists some \( \delta^* < 1 \) high enough such that for all \( \beta \in [\beta_0, 1] \) and \( \delta \in [\delta^*, 1] \) there exist an equilibrium in which the expert’s expected payoff is \( V_e \). Now, fix the value of \( \delta \). Let \( \beta_0 \) be small enough, and so, \( \delta^* \) be high enough so that \( \frac{N_G(1-\delta)}{\delta \pi} \in [\beta_0, 1] \) and \( \delta \in [\delta^*, 1] \). Then, the expert’s expected payoff \( V_e \) takes its maximum value over \( [\beta_0, 1] \) at \( \beta = \frac{N_G(1-\delta)}{\delta \pi} \). Here
is why: Given the value of $V_e$ in Proposition 3, we have

$$\frac{\partial V_e}{\partial \beta} = \frac{\partial \alpha \beta}{\partial \beta} \left( \frac{v_e \pi}{1 - (1 - \pi \beta) \delta} \right) + \frac{\partial v^d}{\partial \beta} \alpha \beta.$$

Since $\frac{\partial v^d}{\partial \beta} = -\frac{\delta \pi^2 v_e}{(1 - (1 - \pi \beta) \beta)^2}$ and $\frac{\partial \alpha \beta}{\partial \beta} = \frac{\alpha \beta N_G(1 - \delta)}{\beta (1 - (1 - \pi \beta) \delta)^2}$, we have

$$\frac{\partial V_e}{\partial \beta} = \frac{\alpha \beta v_e \pi}{(1 - (1 - \pi \beta) \delta)^2} \left[ \frac{N_G(1 - \delta)}{\beta} - \delta \pi \right].$$

Equating the last equation to 0 gives the value of $\beta$ that maximizes $V_e$.

**Proof of Corollary 2.** For $N_G = 1$, it is easy to calculate the expected number of stages that the expert should be truthful to build his reputation up to $\mu^*$. If $N_G = 1$, then it requires only one stage, where the true state is $u$ and the receivers observe the payoffs, to build reputation. Therefore, the expected number of stages that the expert should be truthful is

$$\sum_{i=0}^{\infty} (i + 1)(1 - \pi \beta)^i \pi \beta = \frac{1}{\pi \beta},$$

where the $(i + 1)^{th}$ stage is the first stage in which the true state is $u$ and the receiver observes the realized payoffs, and $(1 - \pi \beta)^i \pi \beta$ is the probability of this event. For arbitrarily large but finite $N_G$, we inductively calculate the expected number of stages that the expert should be truthful.

Recall that the expert can deceive the receivers in equilibrium only if the expert’s reputation is higher than $\mu^*$. According to the equilibrium strategies where the expert’s payoff is the highest, the expert can deceive the receiver at only one stage when $\beta = 1$. However, for smaller values of $\beta$, the expert can deceive the receiver as long as the receivers do not observe the previous deceptions. According to these equilibrium strategies, the probability that the expert deceives the receiver during the entire repeated sender-receiver game only once is

$$\sum_{i=0}^{\infty} \pi (1 - \pi)^i \beta = \beta.$$

Similarly, the probability that the expert deceives the receiver during the entire repeated sender-receiver game only twice is

$$\sum_{i=0}^{\infty} (i + 1) \pi^2 (1 - \pi)^i (1 - \beta) \beta = (1 - \beta) \beta.$$
Inductively, we can find that the probability that the expert deceives only \( n \) times is

\[
\sum_{i=n}^{\infty} \binom{i-1}{n-1} \pi^n (1-\pi)^{i-n+1} (1-\beta)^{n-1}\beta = (1-\beta)^{n-1}\beta.
\]

Hence, in equilibrium, the expected number stages that the expert deceives the receivers is at most

\[
\sum_{i=0}^{\infty} (i+1)(1-\beta)^i\beta = \frac{1}{\beta}.
\]

Proof of Proposition 4. The following strategies form equilibrium of the repeated sender-receiver game where the expert deceives the receiver for an unlimited periods. Let \( M \) be the smallest natural number satisfying

\[
M \geq \ln \left[ \frac{1 - \frac{(1-\delta)v_u}{\delta(1-\pi)v_n}}{\ln \delta} \right].
\]  

(8)

Note that \( M \) is well defined for all \( \delta \geq \delta^* \equiv \frac{v_n}{v_n + (1-\pi)v_u} \). The equilibrium strategies consist of three phases; deception, rewarding and punishment phases. Players start in the deception phase, where the expert sends the message \( n \) regardless of the true state. Deception phase ends when the expert deceives the receiver for a period. Once the deception phase ends, the players move to the rewarding phase and stay there for \( M \) periods. In the rewarding phase the expert reports the state truthfully. Once the rewarding phase ends, both players move to the deception phase again. Therefore, the deception phase, followed by the rewarding phase, repeats indefinitely. In both deception and rewarding phases, the receiver approves (reject) the treatment whenever she observes the message \( n \) (\( u \)). If at least one of the players deviates from his/her strategies in any phase, then the players move to the punishment phase and stay there forever. In the punishment phase, the receiver and the expert play their babbling equilibrium strategies.

We already know that the punishment phase strategies are optimal. The receiver has no incentive to deviate in the rewarding phase. Furthermore, for sufficiently large values of \( \delta \), the expert’s rewarding phase strategies are also optimal: If the expert deviates and sends the message \( n \) when the true state is \( u \), then his continuation payoff is only \( v_c \). However, if he does not deviate, then his continuation payoff is higher than his fully-revealing equilibrium payoff \( \frac{(1-\pi)v_u}{1-\delta} \), which is higher than \( v_c \) for sufficiently high values of \( \delta \).

Therefore, all we need to show is that the deception phase strategies are also optimal. Given the strategies described above, the expert’s continuation payoff in the deception phase is strictly positive. However, his best deviation payoff is 0. Therefore, deviation
is never optimal for the expert in the deception phase. Define $V_r$ to be the receiver’s continuation payoff in any subgame that begins with the deception phase. Likewise, let $W_r$ denote the receiver’s continuation payoff in any subgame that begins with the beginning of the rewarding phase. Therefore,

$$V_r = \pi[-v_u + \delta W_r] + (1 - \pi)[v_n + V_r],$$

and

$$W_r = \frac{(1 - \pi)v_n(1 - \delta M)}{1 - \delta} + \delta M V_r.$$

Solving these two equalities for $V_r$ yields

$$V_r[1 - (1 - \pi + \delta M \pi)\delta] = (1 - \pi)v_n - \pi v_u + \delta \pi \frac{(1 - \pi)v_n(1 - \delta M)}{1 - \delta}.$$ \(\text{(9)}\)

If the receiver deviates in the deception phase and rejects the treatment, then her continuation payoff is simply 0. However, if she approves the treatment, then her continuation payoff is $\pi(-v_u + \delta W_r) + (1 - \pi)(v_n + \delta V_r)$, which is equal to $V_r$.

Note that the left hand side of the equality \(\text{(9)}\) is always positive. To show that $V_r$ is positive, all we need to show that the right hand side of \(\text{(9)}\) is also positive. Because $M$ satisfies the inequality \(\text{(8)}\), it is true that $\delta \frac{(1 - \pi)v_n(1 - \delta M)}{1 - \delta} - v_u \geq 0$. Therefore, the right hand side of \(\text{(9)}\) is positive, implying that $V_r \geq 0$. Hence, the receiver has no incentive to deviate in the deception phase either. This completes the proof.

References


