What Shifts the Beveridge Curve?
Recruiting Intensity and Financial Shocks*

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Abstract
Labor market data show a substantial deterioration of aggregate matching efficiency around the Great Recession, even after controlling for compositional changes among job seekers. We augment the multiworker-firm version of the equilibrium random-matching model of the labor market with endogenous firm entry and exit, a choice of recruiting intensity when hiring, and a dividend constraint that induces some firms to borrow and some of those with debt to default. We use the model to study whether aggregate financial shocks can account for the observed drop in matching efficiency—and the ensuing shift in the Beveridge curve—through a reduction in the average recruiting intensity in the economy. Central to this mechanism is the role of young firms which contribute disproportionately to job creation, display the highest recruitment effort per vacancy and, at the same time, are more dependant on external finance.

Keywords: Aggregate Matching Efficiency, Beveridge Curve, Financial Shocks, Recruiting Intensity, Unemployment, Vacancies, Young Firms.

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1 Introduction

The Beveridge curve describes the empirical relationship between the unemployment rate and the job vacancy rate of an economy. One of the key stylized facts of macroeconomic fluctuations is the very strong negative correlation, close to $-0.9$, between unemployment and vacancies at business cycle frequencies. At the same time, outward shifts of this negatively sloped locus are not uncommon in the historical experience of developed economies (Elsby and Michaels, 2013).

An outward shift of the Beveridge curve attracts a lot of attention from economists and policymakers because, seen through the lenses of the standard Diamond-Mortensen-Pissarides (DMP) model of the labor market, it may indicate a deterioration in the degree of matching efficiency of the labor market. One of the building blocks of the DMP model is the aggregate matching function, a production function that takes the total number of job-seekers and the total number of vacant positions open for recruiting as inputs, and the flow of new hires as output. Matching efficiency is a multiplicative shifter of this production function of hires, akin to total factor productivity in production theory. A decline in matching efficiency means that the labor market is less effective in its fundamental role of connecting idle labor with idle jobs.

The last historical episode of outward movement in the Beveridge curve—and one of the most significant for magnitude and duration—occurred in the wake of the Great Recession of 2007-2009. In April 2012, approximately four years after the onset of the recession, the U.S. job openings rate returned to its level of April 2008, after dropping by 50 percent. However, at the same time, the unemployment rate was still three percentage points higher than in April 2008 (Figure 1). In this paper, in line with standard theory, we interpret this recent shift as a drop in the aggregate matching efficiency of the labor market and we investigate its causes.

We begin with a measurement exercise that estimates a time series for aggregate matching efficiency, a variable that in the rest of the paper we label $\Phi_t$. A recent literature (Veracierto, 2011; Barnichon and Figura, 2010; Shimer, 2012; Fujita and Moscarini, 2013; Hall and Schulhofer-Wohl, 2013) has pointed out that compositional changes in the pool of job-seekers—some of them more effective than others in finding employment—can explain a

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1See Elsby, Hobijn, and Şahin (2010) for a comprehensive account of the U.S. labor market in the aftermath of the Great Recession.
2Recently, Christiano, Eichenbaum, and Trabandt (2013) argued that one does not need any drop in the efficiency parameter of the aggregate matching function to reproduce the joint dynamics of unemployment and vacancies around the Great Recession. We discuss their approach and their findings in Section 2.
sizable fraction of movements in measured $\Phi_t$, very much like the quality composition of labor input accounts for changes in TFP, when the latter is estimated as a “Solow residual” with standard measures of inputs. We build heavily on this literature by including employed and out-of-the-labor-force job-seekers in the matching function, and by excluding from the input and the output of the matching function workers on and hires from temporary layoff. Temporary layoffs are associated with a very high recall rate to the same employer, and hence escape the informational frictions and the heterogeneity that the matching function is designed to capture (Fujita and Moscarini, 2013). The resulting time series for $\Phi_t$, after these corrections, drops by 30-35 percentage points during the last recession and recovers only at a very slow pace since then. To put this magnitude in context, one should keep in mind that, around the 2001 recession, our estimate of $\Phi_t$ falls by only 10 percentage points and rebounds much more quickly.

Our proposed explanation for this sharp deterioration of aggregate matching efficiency in the aftermath of the Great Recession builds on three stylized facts relating to firm dynamics and job creation. First, as Haltiwanger, Jarmin, and Miranda (2012) document, young firms display rich “up-or-out” dynamics: the survival rate of young firms is lower than for older incumbents but, conditional on survival, young firms grow more rapidly than their mature counterparts. As a result, young firms play a critical role in the creation of new jobs. Start-ups account for close to 20 percent of job creation, in spite of representing only three percent of total employment, and the share of firms younger than 5 year-old in total job creation is over one-third.

\footnote{We confirm the importance of the “compositional factor” that, in our calculations, explains another 15-20 percent drop in measured matching efficiency over the same period.}
Second, the job-filling rate—the speed at which a firm fills an open position—displays a lot of heterogeneity in the cross-section of firms. Namely, it increases steeply with employer growth rates (Davis, Faberman, and Haltiwanger, 2013): firms that grow faster also choose to recruit with higher intensity. For example, the job-filling rate almost doubles as monthly employment growth increases from 10 to 20 percent. Presumably, this occurs because these firms spend more in recruitment activities (e.g., through more advertising, higher search effort per vacancy, better screening of applicants, more attractive compensation and working conditions) for vacant position. Since the the input of the aggregate matching function from the firm side is measured through the total number of vacancies, but not through the average recruiting intensity of the hiring firms, fluctuations in this factor would show up in the data as movements in aggregate matching efficiency. This is precisely the channel that we explore in this paper.

Third, it is well established that young, small, and fast-growing firms are those that are most dependent on external finance and on credit market conditions (see, e.g., Hubbard, 1998, for an early survey). In particular, as the Kauffman Firm Survey documents, one important source of funding for start-ups is collateralized borrowing against home equity. Thus, adverse shocks to financial intermediaries and to the value of housing, such as the ones witnessed during the last recession, have an especially fierce impact on young firms. In the context of the Great Recession, the evidence on the importance of this channel is abundant. Chodorow-Reich (2014) exploits the property that bank-borrower relationships are sticky and the fact that, before 2007, major lenders had different degrees of exposure to Lehman Brothers to isolate the exogenous component of the credit shock that hit the U.S. economy. He concludes that the withdrawal of credit accounts for at least 1/3 of the employment decline at small and medium-sized firms. At the same time, he cannot reject the null hypothesis of no effect on the largest firms with access to the bond market. Siemer (2013) uses a difference-in-differences approach to identify the heterogeneous effect of the recession on firms belonging to sectors with various degree of external financial dependence. His results imply that external financial constraints account for a 5-10 percentage point reduction of employment growth in small firms relative to large firms during 2007-2009—a result driven predominantly by young firms. Adelino, Schoar, and Severino (2013) and Fort, Haltiwanger, Jarmin, and Miranda (2013) find that a decline in housing prices in a geographical area yields a significant reduction in the differential net job creation rate be-
tween start-ups/young and mature businesses in that region.\footnote{These authors argue that this effect is a symptom of a collapse of collateralized lending to start-ups, and not just a manifestation of deleveraging, as in \textit{Mian and Sufi} (2011). See also \textit{Mehrotra and Sergeyev} (2013) for an IV approach to isolate the effect of the fall in housing prices on local job creation rates.}

Figure 2 offers a refinement of these facts. By using data from Census’ Business Dynamic Survey (BDS), we are able to disentangle the relative role of firm age and size in aggregate employment dynamics, and to distinguish between the intensive (employment per firm) and the extensive (number of firms) margin of adjustment.

Panel 2a shows that young rather than small firms exhibited the largest relative decline in employment since 2007. Decomposing changes in total employment into employment per firm and number of firms we find that the bulk of the contraction in total employment is explained by the sharp (close to 30 percent) and persistent drop in the number of entrants. Panel 2b shows that this feature is unprecedented in the last two decades. Up to 2007, changes in employment per firm explains nearly all of the cyclical fluctuations in aggregate employment. However post-2007 more than two-thirds of the decline in aggregate employment away from its pre-recession trend is accounted for by the decrease in the number of firms and less than a third by employment-per-firm. Overall, these results are consistent with the notion that young firms growth was most constrained in the crisis and that the shock to the economy was one which impeded firm entry.

Taken together, these three facts suggest a novel narrative for the observed deterioration in aggregate matching efficiency and the ensuing shift in the Beveridge curve that occurred throughout the Great Recession and beyond: an aggregate financial shock hits the economy and curbs labor demand —disproportionately so for start-ups and young firms; since these are the the firms displaying the highest recruiting effort and, at the same time, they account for a substantial share of new vacancies, average recruiting intensity in the economy falls. Through the lens of an aggregate matching function that takes vacancies and unemployment as inputs, the decline in recruiting intensity assumes the form of a drop in measured matching efficiency.

In this paper, we develop a structural equilibrium model to explore the quantitative relevance of this mechanism. Our model is a version of the canonical Diamond-Mortensen-Pissarides random matching framework with decreasing returns in production and non-convex hiring costs (\textit{Cooper, Haltiwanger, and Willis}, 2007; \textit{Elsby and Michaels}, 2013; \textit{Elsby}, 2013; \textit{Elsby and Michaels}, 2013; \textit{Elsby and Michaels}, 2013).
(a) Percentage changes since 2007 by age and size

![Graphs showing percentage changes in employment, employment per firm, and firms by age and size from 2003 to 2011.]

**Note** Data from BDS. Entrant firms are those that are age zero in a given survey year. All decompositions are percent changes from levels in 2007.

(b) Aggregate business-cycle fluctuations

![Graph showing aggregate business-cycle fluctuations from 1986 to 2012.]

**Note** Data from BDS. Series are residuals from a linear regressions of the variable in logs on a linear time trend from 1986 to 2007.

Figure 2: Decomposing employment into employment per firm and number of firms
Acemoglu and Hawkins, Forthcoming). The model simultaneously features a realistic firm lifecycle, as its classic competitive setting counterparts (Jovanovic, 1982; Hopenhayn, 1992), and a frictional labor market with slack on both demand and supply sides—a necessary ingredient of our analysis. We augment this environment in three dimensions. First, we introduce financial frictions: firms face a nonnegativity constraint on dividends that induces some firms to borrow and, some of those with debt, to default. Loan contracts are priced competitively to reflect default risk, in a similar vein to the modelling of financial intermediation in Eaton and Gersovitz (1981) for sovereign debt, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for household debt, and Khan, Senga, and Thomas (2014) for firm debt.5

Second, we allow for endogenous entry and exit of firms. This is a key element because our mechanism hinges on the severity of the shock for new entrants, a fact that—as documented in Figure 2—is well supported empirically. A number of papers has focused on explaining the “missing generation of entrants” in the wake of the Great Recession: Siemer (2013), Schott (2013), and Sedláček (2014), among others, study the impact of financial shocks on the entry of new firms and the long-lasting effects of this missing generation on employment and unemployment, but the implications for aggregate matching efficiency remain unexplored.

Finally, we introduce a firm’s decision on recruiting intensity: hiring firms choose the maximum number of open positions that they are willing to fill in each period, and the amount of resources that they devote to recruitment activities. The sum of all individual firms’ recruitment efforts, weighted by their vacancy share, aggregates to the economy’s measured matching efficiency. To our knowledge, Kaas and Kircher (2011) is the only other paper that embeds recruiting intensity in a structural equilibrium model of the labor market. It differs from our paper because it is a directed search environment where some of the heterogeneity in job-filling rates across firms derives from the different wages firms optimally post to attract jobseekers.6 These authors do not use their model to account for the observed recent drop in matching efficiency. While their framework is flexible enough to investigate the role of productivity shocks, it does not yet incorporate financial shocks.

5 An alternative approach to adding financial frictions to this class of models introduces a search friction in the financial market as well. See, e.g., Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2013).

6 However, in Kaas and Kircher (2011), quantitatively, most of the heterogeneity needed to explain the Davis, Faberman, and Haltianger (2013)’s empirical finding, derives from the specific shape of the labor adjustment cost function. The final functional form adopted in their paper is the same we advocate in Section 3. We have arrived at this conclusion independently.
We parameterize the model in order to match a number of aggregate labor market statistics and firm-level cross-sectional moments. In particular, we choose the key parameters of the recruiting cost function so that the model replicates the empirical relation between the job-filling rate and the growth rate documented by Davis, Faberman, and Haltiwanger (2013) from micro data. The model reproduces firm life-cycles broadly consistent with the data: in particular, the “up or out” dynamics of young firms emphasized by Haltiwanger (2011).

We then ask whether the model is able to account for the dynamics of aggregate match efficiency, and other key labor market variables, during the 2007-09 recession and throughout the slump that followed. To put our findings in context, we contrast the predictions of the model for the Great Recession to those for the 2001 downturn. In our first experiment, we simulate a recession entirely driven by a decline in TFP and think of it as the 2001 recession. In our second experiment, we pair the same decline in TFP with a reduction in the fraction of the start-up cost that entrants can obtain through equity. This second shock translates into a fall in firm entry. A comparison between the two experiments reveals that the financial shock amplifies the fluctuations in vacancies and unemployment and induces a larger and, especially, more persistent drop in aggregate matching efficiency.

The rest of the paper is organized as follows. In section 2 we measure the magnitude of the drop in aggregate matching efficiency in the aftermath of the Great Recession and, as much as the data allow, we compare it to the 2001 recession. Section 3 outlines the model economy and the stationary equilibrium. Section 4 describes the parameterization of the model, and highlights some cross-sectional features of the economy. Section 5 contains the experiments where we trace the dynamic response of the economy to various shocks and outlines the main results of the paper. Section 6 concludes the paper.

\section{Measurement of Aggregate Matching Efficiency}

As a starting point, we stipulate the aggregate matching function

\[ H_t = \Phi_t V_t^\alpha U_t^{1-\alpha}. \] (1)

where \( H_t \) are aggregate hires, \( V_t \) vacancies, \( U_t \) unemployment, and \( \Phi_t \) is aggregate matching efficiency, the object of interest. Throughout this measurement exercise, we focus on the
sample period 2001:1-2014:2 with monthly frequency. We obtain hires and vacancies from the
Job Openings and Labor Turnover Survey (JOLTS), and unemployment from the Current Pop-
ulation Survey (CPS). In the tradition of the measurement of TFP of the aggregate production
function, we obtain $\Phi_t$ as a residual, i.e., we fix the elasticity parameter $\alpha$ and derive $\Phi_t$ from
the ratio $H_t/(V_\alpha^t U^{1-\alpha})$, month by month.

Our model of Section 3 abstracts from heterogeneity in the pool of job-seekers and, thus,
from changes in its composition that may affect measured $\Phi_t$ from equation (1). Recently,
a growing literature has convincingly argued that compositional changes in the pool of job-
seekers account for a sizable fraction of the dynamics of matching efficiency (Veracierto, 2011;
Barnichon and Figura, 2010; Fujita and Moscarini, 2013; Hall and Schulhofer-Wohl, 2013). In
this section, we build on this literature to identify the component of aggregate matching effi-
ciency that is unexplained by the composition of the stock of job-seekers and, in the rest of the
paper, we use our model to study how firms’ choices of recruiting intensity over the business
cycle contribute to the dynamics of this component.

Our first step is to control for the key sources of heterogeneity in the unemployment pool.
Fujita and Moscarini (2013) and Hall and Schulhofer-Wohl (2013) show that, once the unem-
ployed are classified based on the reason of entry into the pool, those on temporary layoff stand
out from the rest in terms of their re-employment probability—mainly because of the high re-
call rate from their previous employer in the first month or two of joblessness. The aggregate
matching function view of labor market frictions—that we maintain throughout this paper—
relies on the presumption that all hires are the outcome of a costly search process on both sides
of the market. However, recalls eliminate search frictions altogether: workers on temporary
layoff do not appear to search for other jobs and their recall does not require opening a new
vacancy. Hence, we follow Fujita and Moscarini’s approach, and exclude these workers from
the argument $U_t$ of the matching function. Symmetrically, we net out from aggregate hires $H_t$
those that originate from workers on layoff.\footnote{We obtain the number of unemployed workers on layoff from the BLS. Both Fujita and Moscarini (2013) and Hall and Schulhofer-Wohl (2013) estimate from the CPS a monthly recall rate for this class of workers around 0.4, and fairly stable since 2001. We use this rate to estimate the number of hires from unemployed workers on layoff that we subtract from $H_t$.}

As Hall and Schulhofer-Wohl (2013) emphasize, over 40 percent of total hires originate from
out of the labor force ($N_t$) and almost as many from employment ($E_t$). To take these additional
categories of job-seekers into account, we generalize our matching function as follows:

\[ H_t = \Phi_t \cdot \left( 1 + s_t^N \frac{N_t}{U_t} + s_t^E \frac{E_t}{U_t} \right)^{1-\alpha} \cdot V_t^\alpha U_t^{1-\alpha}, \]  

where \( s_t^N \) is the fraction of out-of-the labor force job seekers (or, equivalently, the average search intensity of nonparticipants relative to that of unemployed job seekers). Similarly, \( s_t^E \) is the fraction of employed job seekers. Veracierto (2011) shows that, by exploiting the constant-return-to-scale property of the matching function, there is a simple way to identify \( s_t^N \) and \( s_t^E \), i.e.,

\[ s_t^N = \frac{NE_t/N_t}{UE_t/U_t}, \quad \text{and} \quad s_t^E = \frac{EE_t/E_t}{UE_t/U_t} \]  

and, hence, by using data on out-of-the labor force to employment \((NE_t)\) and employment to employment \((EE_t)\) flows from the CPS, we obtain estimates of \( s_t^N \) and \( s_t^E \). For consistency, also in these formulas \( UE_t \) is measured net of transitions from temporary layoff into employment, and \( U_t \) net of jobless workers on layoff. Finally, as standard in this literature, we set \( \alpha = 0.5 \).

Figure 3 plots both the composition factor in parenthesis in equation (2)—directly measured from the data—and the residual component \( \Phi_t \). The log scale makes the two components additive. Overall, the composition factor explains half of the drop in total matching efficiency during the 2001 recession and 1/3 of over the two years 2007-2009. We identify a fall in \( \Phi_t \)—our object of interest—of around 10 percentage points during the 2001 recession and around 35 percentage points during the 2007-09 downturn. The decline in \( \Phi_t \) is a lot more persistent in the last recession. Five years after the start of the 2001 downturn, \( \Phi_t \) is back to its pre-recession level, whereas five years after the Great Recession, \( \Phi_t \) is still 20 percent lower.

In conclusion, our analysis shows that, even after broadly controlling for compositional changes among job seekers, there is a sizable drop in aggregate matching efficiency that remains to be explained. Clearly, one cannot rule out a common reduction in unobserved search intensity across all different types of job seekers: such phenomenon would be observationally equivalent to a lower value for \( \Phi_t \). However, the available evidence points to the opposite di-

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8For job-to-job transitions \( EE_t \), we use the series initially compiled by Fallick and Fleischman (2004), and recently updated.

9Also a rise in mismatch between unemployment and vacancies across labor markets would translate into
reduction. Mukoyama, Patterson, and Şahin (2013) construct a monthly series of aggregate worker search effort by combining information from the American Time Use Survey (ATUS) and the CPS. Their main finding is that search effort is markedly countercyclical.\footnote{This finding may appear surprising, in light of the generous and repeated extensions of UI benefits implemented by the U.S. government during the post-recession slump. However, the existing evidence seems to point towards the fact that, in order to find large effects of UI benefits on unemployment, one has to resort to its equilibrium impact on job creation (Hagedorn, Karahan, Manovskii, and Mitman, 2013). If that is the case, more generous UI compensation would show up in lower vacancies, not in lower matching efficiency.}

\subsection{2.1 Relationship to Christiano, Eichenbaum and Trabandt}

In a recent influential paper, Christiano, Eichenbaum, and Trabandt (2013), henceforth CET, argue that one can account for the apparent shift of the Beveridge curve during the Great Recession without any drop in aggregate matching efficiency. They emphasize that the key step is to fully incorporate dynamics into the equilibrium equation for the Beveridge curve, instead of imposing steady-state at every date $t$. In a two-state model of the labor market, without on-the-job search, the law of motion for unemployment, together with the matching function, yields the a drop in $\Phi_t$. According to Sahin, Song, Topa, and Violante (2014), the quantitative importance of this channel around the Great Recession was limited.

Figure 3: Measured aggregate matching efficiency $\Phi_t$ and composition factor. Sample period: 2001:2-2014:4.
Beveridge curve equation:

\[ U_{t+1} = U_t - \Phi_t V_t^\alpha U_t^{1-\alpha} + \delta (1 - U_t), \]  

where \( \delta \) is the separation rate. It is also useful to define the job-finding rate

\[ \frac{H_t}{U_t} = \Phi_t \left( \frac{V_t}{U_t} \right)^\alpha = \Phi_t \theta_t^\alpha, \]  

and the vacancy yield (or job-filling rate)

\[ \frac{H_t}{V_t} = \Phi_t \left( \frac{U_t}{V_t} \right)^{1-\alpha} = \Phi_t \theta_t^{\alpha-1}. \]

We now run an exercise in the spirit of CET. We use monthly data on unemployment \( U_t \) and hires \( H_t \)—again, for consistency, net of the temporary layoff components, and the latter also divided by the composition factor—set \( \delta = 0.033 \) and \( \alpha = 0.5 \), as in CET, and fix \( \Phi_t \) to a constant in the three equations (4)-(6). We then obtain residually vacancies \( \hat{V}_t \) from equation (4) and trace the Beveridge curve in the U-V space. Through this exercise, CET aim to show that, if a model is able to generate equilibrium unemployment dynamics similar to those in the data, and equation (4) is one of its equilibrium conditions, then the same model will necessarily be able to explain the shift in the Beveridge curve.\(^{11}\)

Figure 4 displays the findings of this exercise. The top-left panel shows that, even in absence of a drop in \( \Phi_t \), equation (4) can produce an outward shift of similar magnitude to the empirical shift. A more careful analysis, though, reveals that the Beveridge curve relation appears too steep relative to the data. In other words, vacancies \( \hat{V}_t \) implied by (4) with constant \( \Phi_t \) are extremely volatile: for a given change in unemployment observed in the data, vacancies move a lot more than in the data.\(^{12}\) The vacancy yield plotted in the bottom-left panel confirms this impression. The model’s vacancy yield rises twice as much as in data during the Great Recession because of the exaggerated fall in \( \hat{V}_t \).

Allowing for a deterioration of matching efficiency during 2008-2009 helps limiting the ex-

\(^{11}\)The presentation slides in CET—but not the published paper—contain this exercise and report almost identical results.

\(^{12}\)Not surprisingly, the loop in the Beveridge curve implied by the full model in CET (their Figure 9) is a lot wider than in the data.
Figure 4: Top-left panel: Beveridge curve in the data and as implied by equation (4) with $\Phi_t$ constant. Top-right: job-finding rate. Bottom-left: vacancy yield with $\Phi_t$ constant. Bottom-right: vacancy yield with $\Phi_t$ varying as estimated in Section 2.

The model-implied job finding rate is, by construction, identical in the two exercises. To clarify this point, we perform the same exercise, but this time we also feed our estimated path for $\Phi_t$ into equations (4)-(6). Figure 4 collects our findings. The model now matches the vacancy yield very well.\textsuperscript{13}

\subsection*{2.2 Recruiting Intensity and Aggregate Matching Efficiency}

We now describe briefly how, starting from individual hiring decisions at the firm level, we can aggregate into an economy-wide matching function whose efficiency factor has the interpretation of average recruitment intensity in the economy. This derivation follows

\textsuperscript{13}The model-implied job finding rate is, by construction, identical in the two exercises.
Davis, Faberman, and Haltiwanger (2013), henceforth DFH.

Any given hiring firm $i$ chooses $v_{it}$, the maximum number of open positions, ready to be staffed and costly to create, and $e_{it} \in [0, 1]$, recruiting intensity — the probability of filling each open position. Let $v^*_{it} = e_{it}v_{it}$ be the number of effective vacancies in firm $i$. Integrating over all firms we arrive at

$$V^*_t = \int e_{it}v_{it}di,$$

(7)

the aggregate number of effective vacancies. Then, under our maintained assumption of a CRS Cobb-Douglas matching function, aggregate hires are given by

$$H_t = (V^*_t)^\alpha U^{1-\alpha}_t = \Phi_t V^*_t U^{1-\alpha}_t, \quad \text{with} \quad \Phi_t = \left(\frac{V^*_t}{V_t}\right)^\alpha = \left[\int e_{it} \left(\frac{v_{it}}{V_t}\right)\right]^\alpha.$$

(8)

Measured aggregate matching efficiency $\Phi_t$ is, therefore, an average of firm-level recruitment intensity weighted by individual vacancy shares, raised to the power of $\alpha$, the economy-wide elasticity of hires to vacancies.\(^{14}\) Finally, note that consistency requires that, at the individual level, firm $i$ faces hiring frictions that can be summarized as

$$h_{it} = q \left(\theta^*_t\right) e_{it}v_{it},$$

(9)

where $\theta^*_t = V^*_t / U_t$ is effective market tightness.\(^{15}\) Thus, $q \left(\theta^*_t\right) = H_t / V^*_t = (\theta^*_t)^{\alpha-1}$ is the aggregate job filling rate per effective vacancy, constant across all firms at date $t$.

In the rest of the paper, we take the empirical estimate of $\Phi_t$ from Figure 3 as the target series for aggregate matching efficiency, and quantitatively evaluate whether the model-implied movements in aggregate recruiting intensity can explain its dynamics.\(^{16}\)

\(^{14}\)DFH call (8) the generalized matching function.

\(^{15}\)In the paper, we are faithful to the notation in this literature and denote measured labor market tightness $V_t / U_t$ as $\theta$.

\(^{16}\)Since we specify the aggregate matching function as equation (1), for full consistency, we use data on aggregate hires $H_t$ divided by the composition factor in equation (2) as estimated above. Moreover, recall that, when we construct aggregate hires and unemployment from the data, we subtract hires from workers on temporary layoffs from total hires and unemployed workers on temporary layoff from total unemployment. These are the data series that we use whenever we do model-data comparisons on $H_t$ and $U_t$. 

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3 Model

Our starting point is the equilibrium random-matching model of the labor market in which firms are heterogeneous in productivity and size, and the hiring process occurs through an aggregate matching function. As discussed in the Introduction, we augment this model in three dimensions—all of which are essential to develop a framework that can address our question. First, our framework features endogenous firm entry and exit. Second, beyond the number of positions to open (vacancies), hiring firms optimally choose their recruiting intensity: by spending more on recruitment resources, they can increase the rate at which they meet job seekers. Third, firms face a dividend constraint that they can overcome, when binding, by borrowing from banks at a premium that reflects their default risk.

In what follows, we present the economic environment in more detail, we outline the model timing, and then we describe the firm, bank, and household problem. Finally, we define a stationary equilibrium for the aggregate economy. Since our experiments will consist of perfect foresight transition dynamics we do not make reference to aggregate state variables in agents’ problems and use a recursive formulation throughout.

3.1 Environment

Time is discrete and the horizon is infinite. Three types of agents populate the economy: firms, banks, and households.

Firms. There is a measure $\lambda_0$ of potential entrants each period, and the measure of incumbent firms is $\lambda$. Firms are heterogeneous in their productivity $z \in Z$, stochastic and i.i.d. across all firms, and operate a decreasing-returns-to-scale (DRS) production technology whose only input is labor $n \in N$. The output of production is an homogeneous final good, whose competitive price is the numeraire of the economy. When active, firms face a non-negativity constraint on dividends. The model timing, explained in detail below, is such that firms must finance wage bills, operating costs, and hiring costs, and pay dividends to households before receiving the revenues from current period production. Even though revenues can be stored —relaxing the next period dividend constraint— some firms need to borrow from banks to satisfy the current

\[^{17}\text{Whereas } \lambda_0 \text{ is a parameter, } \lambda \text{ is determined endogenously.}\]
period constraint.

Potential entrants draw a value of $z$ from the initial distribution $\Gamma_0(z)$ and, conditional on this draw, decide whether to enter and become an incumbent—an action that requires paying the set-up cost $\chi_0$. Entrant firms receive an initial equity injection from households that covers a fraction $\epsilon$ of the set-up cost and must borrow the rest. This is the only time when firms can obtain funds directly from households—throughout the rest of their lifecycle they must rely on the financial sector.

Incumbents can exit exogenously or endogenously. With probability $\delta$, a destruction shock hits an incumbent firm, forcing it to exit with zero residual value to both shareholders (households) and bondholders (banks). The firms surviving this Poisson shock observe their new value of $z$, drawn from the conditional distribution $\Gamma(z', z)$, and choose whether to exit or to continue production. Upon choosing to exit, firms recover last period’s production. Debt is senior to equity in firms’ capital structure, thus, if this is enough to repay the existing debt, shareholders can claim the residual; if it is not, the bank recovers part of their loan and shareholders get nothing from the liquidated firm. Note that this means that exit can occur either with or without default.

Those incumbents that, after observing $z$, decide to stay in the industry first lose a fraction $\zeta$ of their workers in exogenous quits. They then pay a per-period operating cost $\chi$ and choose whether to fire some of their existing employees or hire new workers. Firing is frictionless, but hiring is not: a hiring firm chooses both vacancies $v$ and recruitment effort $e$ with associated hiring cost $C(e, v, n)$. Given $(e, v)$, the individual hiring function (9) determines next period employment $n'$.

Period-by-period bargaining between every individual worker and the firm determines wage payments. We simplify this block of the model by assuming that the object of bargaining is the static marginal surplus (i.e. surplus for that period only) created by each individual match.

**Banks.** The banking sector is perfectly competitive and has free entry. Banks receive household deposits and extend loans to firms. Both are one-period contracts. Transforming deposits into loans is subject to a cost $\varphi$ (in terms of the final good) per unit of funds intermediated. Deposits earn the risk-free return $\bar{Q}^{-1}$. The unit price of a loan to a firm with productivity $z$, target
Draw $z (n, a, z)$

<table>
<thead>
<tr>
<th>$(n, a, z)$</th>
<th>$\delta$ shock</th>
<th>$\lambda_e$</th>
<th>$v'$</th>
<th>$n'$</th>
<th>$w$</th>
<th>$D, C, T', M'$</th>
<th>$y(z, n')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw $z$</td>
<td>Exit/Default</td>
<td>Entry</td>
<td>Borrow</td>
<td>Hire: $(e, v)$</td>
<td>V-U meet</td>
<td>Payment of prod. costs</td>
<td>Production</td>
</tr>
<tr>
<td></td>
<td>Stay &amp; Repay</td>
<td>$n = 0$</td>
<td>at price</td>
<td>Fire: $n' \leq n$</td>
<td>Bargain</td>
<td>$a = y(z, n') - b'$</td>
<td>and dividends</td>
</tr>
<tr>
<td>$\lambda, \Upsilon$</td>
<td>$a = -(1 - \epsilon)\chi_0$</td>
<td>$Q(n', b', z)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>H'hold consumption</td>
</tr>
</tbody>
</table>

Figure 5: Timeline of the model

employment $n'$, and desired loan size $b' > 0$ is $Q(n', b', z)$, determined by the equilibrium zero-profit condition that holds for each loan type separately. The loan price is a function of all the firm’s state variables that influence the repayment probability next period. In what follows, we use the convention that $Q(n', b', z) = \bar{Q}$ for $b' < 0$ (a deposit). Firms behave competitively in the financial market, and take the price function $Q(\cdot)$ as given.

**Households.** We envision a representative household with $\bar{L}$ family members, $\Upsilon$ of which are unemployed. The representative household is risk-neutral and has discount factor $\beta \in (0, 1)$. It trades shares $M$ of the mutual fund comprised of all firms in the economy and makes bank deposits $T$. It earns interests on deposits, the total wage bill paid by firms to the employed family members, and $D$ dividends per share held in the mutual fund. Moreover, unemployed workers, when not searching, produce $\pi$ units of the final good at home.

Before describing the firm’s problem in detail, we outline the precise timing of the model, summarized in Figure 5. Within a period, the events unfold as follows: (i) realization of the productivity and firm destruction shocks, and exogenous exit of incumbents; (ii) exit/default decision by incumbents and entry decision by potential entrants; (iii) exogenous quits; (iv) new borrowing decisions by incumbents; (v) hiring/firing decisions; (vi) labor market matching and firm-worker bargaining; (vii) payment of operating cost, hiring costs, wage bill, existing debt, and dividends by incumbents and household consumption/saving decisions; (viii) production.

For what follows, it is useful to note that the measures of incumbent firms, employment and unemployment are taken at the beginning of the period, between stages (i) and (ii). Moreover, even though the labor market opens after firms exit or fire, workers who separate in the current period can only start searching in the next one. Finally, since remaining active requires that
the firm repays its outstanding debt, the relevant state variable for the firm’s decisions is post-repayment net-worth \( a = y(z_{-1}, n) - b \in A \), where \( y(z_{-1}, n) \) are previous-period revenues from production, and \( b \) is outstanding debt.

### 3.2 Firm problem

We first consider the entry and exit decisions, and then analyze the problem of incumbent firms.

**Entry.** A potential entrant who has drawn \( z \) from \( \Gamma_0(z) \) solves the following problem

\[
\max \left\{ V^i(0, - (1 - \varepsilon) \chi_0, z) - \varepsilon \chi_0, 0 \right\}, \tag{10}
\]

where \( V^i \) is the value of an incumbent firm, a function of \((n, a, z)\). The firm enters if the value, to the risk-neutral shareholder, of becoming an incumbent with no employment \((n = 0)\), initial net worth \( a = - (1 - \varepsilon) \chi_0 \), and productivity \( z \) exceeds the cost of the initial equity injection \( \varepsilon \chi_0 \). Let \( i(z) \in \{0, 1\} \) be the entry decision rule.\(^{18}\) As \( V^i \) is increasing in \( z \), there is an endogenous productivity cut-off \( z^* \) such that for all \( z \geq z^* \) the firm chooses to enter. Thus, the measure of entrants is

\[
\lambda_e = \lambda_0 \int_Z i(z) d\Gamma_0 = \lambda_0 [1 - \Gamma_0(z^*)]. \tag{11}
\]

**Exit.** After surviving the Poisson destruction shock, an incumbent with \( n \) units of labor, post-revenues net worth \( a \), and current and past productivity \( z \) chooses whether to continue production, exit without defaulting, or exit and default by solving

\[
V(n, a, z) = \max \left\{ V^i((1 - \zeta) n, a, z), 0 \right\}. \tag{12}
\]

Continuing production has value \( V^i \) and \( \zeta \) fraction of workers quit the firm. If net-worth \( a \) is positive then the firm can exit, repay its debt obligations and pay out its net-worth to shareholders as dividends. Defaulting implies exit with a residual value of zero to shareholders. We denote by \( x^R(n, a, z) \in \{0, 1\} \) the exit decision with debt repayment, and by \( x^D(n, a, z) \in \{0, 1\} \) the exit decision with default.

\(^{18}\)Since all the potential entrants share the same entry cost and the same initial employment, the entry decision is only a function of their initial productivity draw.
**Hire or fire.** An incumbent firm \((i)\) with employment, assets, and productivity equal to the triplet \((n, a, z)\) that chooses whether to hire or fire workers solves

\[
V^i(n, a, z) = \max \left\{ V^h(n, a, z), V^f(n, a, z) \right\},
\]

where the two value functions associated with firing \((f)\) and hiring \((h)\) are described below.

**The firing firm.** A firm that has chosen to fire some of its workers (or not to adjust its workforce) solves

\[
V^f(n, a, z) = \max_{n', b'} d^f + \beta(1 - \delta) \int_Z V(n', a', z') \Gamma(dz', z)
\]

s.t.

\[
\begin{align*}
n' & \leq n \\
a' & = y(z, n') - b' \\
d^f & \equiv a - w(n', z)n' - \chi + Q(n', b', z)b' \geq 0
\end{align*}
\]

Firms maximize the shareholder’s value and, because of risk-neutrality, they use \(\beta\) as their discount factor, adjusted by the survival rate \((1 - \delta)\). Dividends \(d^f\) for a firing firm are given by its post-repayment net worth \(a\), net of the wage bill, and operating cost plus the additional resources borrowed from the bank. As it is clear from the last equation in \((14)\), firms face a non-negativity constraint on dividends. We let \(n'_{-}(n, a, z)\) denote the firing decision rule.

Every period, the firm bargains with each worker over the marginal surplus from the ongoing match, with the wage function solving:

\[
w(n, z) = \eta \left[ y_n(n, z) - w_n(n, z)n \right] + (1 - \eta)\pi
\]

where \(\eta\) is the bargaining power of the worker.

**The hiring firm.** The hiring firm chooses the number of vacancies to post \(v \in \mathbb{R}_+\), recruitment effort \(e \in [0, 1]\) and, by a law of large numbers, understands that its new hires \(n' - n\) will be given by the probability that its effective vacancies \(v^* = ve\) meet a worker—equal to \(q\) and taken as given by the firm—times the number of effective vacancies created, or \(n' - n = q(\theta^*)ev\). The firm faces a variable cost function \(C(e, v, n)\), increasing and convex in \(e\) and \(v\).
Note that a firm’s continuation value depends on \( n' \), not on the mix of recruiting intensity \( e \) and vacant positions \( v \) that generates it. As a result, one can split the problem of the hiring firm in two stages. First, the choice of \( n' \). Second, given \( n' \), the choice of the optimal combination of inputs \((e, v)\). This latter problem reduces to a simple static cost-minimization problem:

\[
C^* (n, n') = \min_{e, v} C(e, v, n) \\
\text{s.t.} \\
e \geq 0, \ v \geq 0 \\
n' - n = q(\theta^*)ev
\]

yielding the lowest cost combination \( e(n, n') \) and \( v(n, n') \) that delivers \( h = n' - n \) hires to a firm of size \( n \). Let \( C^* (n, n') \) be the implied cost function, expressed in terms of target employment \( n' \).

The choice of \( n' \) requires solving the dynamic problem

\[
\mathbb{W}^h(n, a, z) = \max_{n', b'} d^h + \beta(1 - \delta) \int_Z \mathbb{W}(n', a', z') \Gamma(dz', z) \\
\text{s.t.} \\
n' > n \\
a' = y(z, n') - b' \\
d^h \equiv a - \xi n' - \chi - C^* (n, n') + Q(n', b', z)b' \geq 0
\]

The solution of this problem is a decision rule \( n'_+ (n, a, z) \). Using this function in the solution of (16), we obtain decision rules \( e(n, a, z) \) and \( v(n, a, z) \) for recruitment effort and vacancies in terms of the current firm states. Finally, recall that, also in this case, dividends \( d^h \) must be non-negative.

Given the centrality of the hiring cost function \( C(e, v, n) \) to our analysis, here we briefly discuss its specification. In what follows, we choose the functional form

\[
C(e, v, n) = \left[ \frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left( \frac{v}{n} \right)^{\gamma_2} \right] v, \tag{18}
\]
with $\gamma_1 \geq 1$ and $\gamma_2 \geq 0$ being the necessary conditions for convexity of $C$.\footnote{In the limiting case $\gamma_1 = 1, \gamma_2 = 0$, the model collapses to the standard matching model without recruiting intensity margin: the optimal effort choice is 1 and the job filling rate is equal across all firms.} This cost function implies that the average cost of a vacancy, $C/v$, has two separate components. The first one is increasing in recruiting intensity per vacancy $e$. The idea is that for any given open position, the firm can choose to spend resources of recruitment activity to make the position more visible or the firm more attractive, to assess more candidates or to better screen them, but all such activities are costly. The second component is the vacancy rate, and it captures the fact that expanding productive capacity is costly in relative terms: the implicit presumption is that, for example, creating 10 new positions involves a more expensive reorganization of production in a firm with 10 employees than in a firm with 1000 employees.\footnote{When we solve the model, in order to avoid division by zero for new entrants, we make a small adjustment to the specification in (18): we write the vacancy rate in the second term as $v/(n + n_0)$ and set $n_0 = 1$. One rationale for this correction is that even the smallest start-up has at least an entrepreneur to begin with. Kaas and Kircher (2011) make a similar adjustment.}

In Appendix A we derive a number of results for the static hiring problem of the firm under this cost function and derive the exact expression for $C^*(n, n')$ used in the dynamic problem. We show that, by combining first-order conditions, we obtain the optimal choice of $e$ and, hence, the firm-level job filling rate:

$$f(n, n') \equiv q(\theta^*) e(n, n') = \left[ \frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*) \frac{\gamma_1}{\gamma_1 + \gamma_2} \left( \frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}. \quad (19)$$

This equation demonstrates that the model implies a constant-elasticity log-linear relation between the job filling rate and employment growth at the firm level. This is the key empirical finding of DFH. In fact, one could interpret our functional choice for $C$ in equation (18) as a “reverse-engineering” strategy in order to obtain, from first principles, the empirical cross-sectional relation between firm-level job-filling rate and firm-level hiring rate uncovered by DFH. Put differently, micro data sharply discipline the recruiting cost function of the model.

Appendix A also shows that, once (18) includes the optimal choice of $e$, $C$ is equivalent to the hiring cost function assumed by Kaas and Kircher (2011).

Why does firm optimality imply that the job filling rate increases with the growth rate with elasticity $\gamma_2 / (\gamma_1 + \gamma_2)$? For two reasons. First, recruiting intensity and the vacancy rate ($v/n$) are complements in the production of the hires per employee $(n' - n)/n$, the firm’s growth
rate—see the last equation in (16). Thus, a firm that wants to grow a lot will optimally create more positions and, at the same time, spend more in recruiting effort. Second, the stronger the convexity of $C$ in the vacancy rate ($\gamma_2$), relative to its degree of convexity in effort ($\gamma_1$), the more an expanding firm finds it optimal to substitute away from vacancies into recruiting intensity to realize its target growth rate. In the special case when $\gamma_2 = 0$, recruiting effort is irresponsive to the growth rate, as in Pissarides (2000).

### 3.3 Bank problem

A firm approaching a bank for a loan of size $b'$ has committed to its employment level $n'$, which the bank observes along with the firm’s current productivity level $z$. Thus, the firm’s type when borrowing is $(n', b', z)$. The free entry condition induces equality between the expected return on the loan and the risk-free return $\bar{Q}^{-1}$ paid on deposits, thereby determining the unit price $Q(n', b', z)$ of the loan extended to such a firm. Note that, since the household is risk-neutral and infinitely lived, it is easy to anticipate that $\bar{Q} = \beta$ in equilibrium.

The expected return on the loan has to be consistent with the repayment decision of the firm, $1 - x_D(n', a', z')$: the firm repays whenever it does not default ($x_D(\cdot) = 0$). Recall that, upon endogenous default, the bank can recover last period output $y(z, n')$, whereas when the Poisson destruction shock strikes, with probability $\delta$, all assets of the firm are lost.

Combining all these features, the following pricing rule obtains:

$$Q(n', b', z) = (1 - \varphi)\bar{Q}(1 - \delta) \left[1 - \int_Z x_D(n', y(z, n') - b', z') \left(1 - \frac{y(z, n')}{b'}\right) \Gamma(dz', z)\right], \quad (20)$$

where in the second argument of $x_D$ we have used the relationship between post-revenue net worth and debt $a' = y(z', n) - b'$. Even in absence of endogenous default, the loan premium $\bar{Q}/Q(\cdot)$ is positive and equal to $[(1 - \varphi)(1 - \delta)]^{-1}$ because of the intermediation cost and the exogenous firm destruction. Finally, when $\varphi > 0$, the firm always finds borrowing costly relative to its discount factor $\beta(1 - \delta)$. This will induce a precautionary saving motive that kicks in as the firm approaches its (frictionless) optimal size.
3.4 Household problem

The representative household solves

$$W(U, T, M) = \max_{T', M', C} \quad C + \beta W(U', T', X')$$

s.t.

$$C > 0, \quad M' \leq 1$$

$$C + \bar{Q}T' + PM' = \int w(n', z)n' d\lambda + \pi U' + (D + P)M + T$$

$$U' = U - p(\theta^*)U + F(\theta^*)$$

In (21), $C$ denotes household consumption, $U$ denotes the number of unemployed members, $T$ are bank deposits, and $M$ are shares of the mutual fund composed of all firms in the economy, with the aggregate number of shares normalized to one. The share price is $P$ and owning shares entitles the household to dividends $D$, the aggregate of all firms’ dividends.\(^{21}\) The total wage bill is the integral over all wage payments from firms, while workers that are idle this period and begin next period as unemployed job seekers produce $\pi$ units of the final good via home production. Unemployment evolves due to matches at rate $p(\theta^*)$ and separations of mass $F(\theta^*)$ which the household takes as given and we specify later.

From the first-order conditions for deposits and share holdings we obtain $\bar{Q} = \beta$ and $P = \beta (P + D)$ which imply a constant return of $\beta^{-1}$ on both deposits and shares and, thus, the household is indifferent over portfolios. Since the household is risk neutral, it is also indifferent over the timing of consumption.

3.5 Stationary equilibrium and aggregation

Let $\Sigma_N$, $\Sigma_A$, and $\Sigma_Z$ be the Borel sigma algebras over $N$ and $A$, and $Z$. The state space for an incumbent firm is $S = N \times A \times Z$, and we denote with $s$ one of its points $(n, a, z)$. Let $\Sigma_S$ be the sigma algebra on the state space, with typical set $S = N' \times A \times Z$, and $(S, \Sigma_S)$ be the corresponding measurable space. Denote with $\lambda : \Sigma_S \rightarrow [0, 1]$ the stationary distribution of

\(^{21}\)Households consider the initial equity injections into start-ups as negative dividends.
incumbent firms at the beginning of the period, before the exogenous exit shock.\footnote{Note a subtlety in the timing here. We described the values of the firms after the exogenous exit shock, whereas we describe the measure before the shock (see the timeline in Figure 5). This eases the formulation of the laws of motion of aggregate state variables.}

To simplify the exposition of the equilibrium, it is convenient to (i) define the implied decision rule for post-revenue net worth $a' (n, a, z) = y (z, n) - b' (n, a, z)$, the encompassing exit decision rule, including both repayment and default, $x = 1 - (1 - x^R) \cdot (1 - x^D)$, and the encompassing employment rule, including both hiring and firing, $n' (n, a, z) = n_+ (n, a, z)$ if $n' \leq n$ and $n' (n, a, z) = n_0 (n, a, z)$ if $n' > n$; (ii) use $s$ as the argument for incumbents’ decision rules.

A stationary recursive competitive equilibrium is a collection of firms’ decision rules $\{i (z), x^R (s), x^D (s), n_+ (s), n_- (s), e (s), v (s), a' (s)\}$, value functions $\{V^f, V^h, V^i, V^d\}$, a measure of entrants $\lambda_e$, a pricing function for loans $Q (\cdot)$, a price of safe deposits $\bar{Q}$, share price $P$ and aggregate dividends $D$, wage function $w (\cdot)$, a distribution of firms $\lambda$, and a value for effective labor-market tightness $\theta^*$ such that: (i) the decision rules solve the firms problems (10)-(17), $\{V^f, V^h, V^i, V^d\}$ are the associated value functions, and $\lambda_e$ is the mass of entrants at productivity $z$ implied by (11); (ii) the pricing function $Q (\cdot)$ satisfies the free entry condition (20) for every segment $(n', b', z)$; (iii) $\bar{Q} = \beta^{-1}$ from household optimization; (iv) the market for shares clears at $M = 1$ with share price

$$P = (1 - \delta) \int_S V (s) \, d\lambda + \lambda_0 \int_Z V^i (0, - (1 - \epsilon) \chi_0, z) \, i (z) \, d\Gamma_0$$

and dividends

$$D = (1 - \delta) \int_S \left\{ [1 - x (s)] \, d (s) + x^R (s) \, a \right\} \, d\lambda - (\epsilon \chi_0) \lambda_0 \int_Z i (z) \, d\Gamma_0$$

(v) the wage function is given by (15); (vi) the stationary distribution $\lambda$ is the fixed point of the recursion:

$$\lambda (\mathcal{N} \times \mathcal{A} \times \mathcal{Z}) = (1 - \delta) \int_S [1 - x (s)] \mathbf{1}_{\{n' (s) \in \mathcal{N}\}} \mathbf{1}_{\{a' (s) \in \mathcal{A}\}} \Gamma (\mathcal{Z}, z) \, d\lambda$$

$$+ \lambda_0 \int_Z i (z) \mathbf{1}_{\{n' (n_0, - (1 - \epsilon) \chi_0, z) \in \mathcal{N}\}} \mathbf{1}_{\{a' (n_0, - (1 - \epsilon) \chi_0, z) \in \mathcal{A}\}} \Gamma (\mathcal{Z}, z) \, d\Gamma_0,$$
where the first term refers to existing incumbents and the second to new entrants; (v) effective market tightness $\theta^*$ is determined by the balanced flow condition

$$L - N(\theta^*) = \frac{F(\theta^*)}{p(\theta^*)},$$  \hspace{1cm} (24)$$

where $N(\theta^*)$ is aggregate employment by incumbents and entrants,

$$N(\theta^*) = (1 - \delta) \int_S [1 - x(s)] n'(s) d\lambda + \lambda_0 \int_Z i(z) n'(n_0, -(1 - \varepsilon) \chi_0, z) d\Gamma_0$$  \hspace{1cm} (25)$$

and $F(\theta^*)$ are aggregate separations

$$F(\theta^*) = \delta \int_S n d\lambda + (1 - \delta) \int_S x(s) n d\lambda + (1 - \delta) \int_S [1 - x(s)] \left[ \zeta n + ((1 - \zeta) n - n'(s))^{-} \right] d\lambda,$$

which include all the employment lost by firms exiting exogenously and endogenously, exogenous quits from continuing firms, plus all the workers fired by shrinking firms, which we have denoted by $((1 - \zeta)n - n'(s))^{-}$.\footnote{Entrant firms never fire, as they enter with the lowest value on the support for $N, n_0$.}

In the three equations above, the dependence of $N$ and $F$ from $\theta^*$ comes through the decision rules and the stationary distribution, even though, for notational ease, we have omitted $\theta^*$ as their explicit argument.

The left-hand side of (24) is the definition of unemployment—labor force minus employment—whereas the right-hand side is the steady-state Beveridge curve, i.e., the law of motion for unemployment

$$U' = U - p(\theta^*) U + F(\theta^*)$$  \hspace{1cm} (27)$$

in steady-state. Thus, exactly as in Elsby and Michaels (2013), the two sides of (24) are independent equations determining the same variable—unemployment—and, combined, they yield equilibrium market tightness $\theta^*$.\footnote{Our computation showed that, typically, $N(\theta^*)$ is decreasing in its argument and the right-hand side of (24) is always positive and decreasing. Thus, the crossing point of left- and right-hand side is unique, when it exists. However, an equilibrium may not exist. For example, for very low hiring costs, $N(\theta^*)$ may be greater than $\bar{L}$. Conversely, for large enough operating or hiring costs, no firms will enter the economy. In this case, there is no equilibrium with market production (albeit there is always some home-production in the economy).}

Clearly, once $\theta^*$ is determined, so is $U$ from either side of (24) and, therefore, $V^*$. Finally, we
note that measured aggregate matching efficiency, in equilibrium, is:

\[
\Phi = \left[ \frac{(1 - \delta) \int_S [1 - x(s)] e(s) v(s) d\lambda + \lambda_0 \int_Z i(z) e(n_0, -(1 - \epsilon) \chi_0, z) v(n_0, -(1 - \epsilon) \chi_0, z) d\Gamma_0}{V} \right]^{\alpha},
\]

(28)

where measured total vacancies are

\[
V = (1 - \delta) \int_S [1 - x(s)] v(s) d\lambda + \lambda_0 \int_Z i(z) v(0, -(1 - \epsilon) \chi_0, z) d\Gamma_0.
\]

(29)

4 Parameterization

We begin from the subset of parameters that are calibrated externally. The model period is one month. We set \( \beta \) to replicate an annualized risk-free rate of 4%. Since the measure of potential entrants \( \lambda_0 \) scales \( \lambda \)—see equation (23)—we choose \( \lambda_0 \) to normalize the total measure of incumbent firms to one. We set the parameter \( \epsilon \) that measures the fraction of the start-up cost \( \chi_0 \) financed by household equity to 0.50 to replicate the total (insider and outsider) equity share of funding for start-ups, measured in the year 2004 before the nascent businesses—a sample of startups in the Kauffman Firm Survey—made any revenues (Robb and Robinson, 2012, Table 5). To be consistent with the empirical analysis of Section 2, we set the elasticity of aggregate hires to aggregate vacancies in the matching function \( (\alpha) \) to 0.5. We set \( \eta \), the worker’s share of the surplus to 0.5. The wedge \( \varphi \) represents the borrowing premium for firms net of the default risk, i.e., the excess bond premium. Gilchrist and Zakrajšek (2012) estimate this excess bond premium to be roughly 2 percent per year before the onset of the 2007 recession. Table 1 summarizes these parameter values.

This same table lists the remaining 16 parameters of the model that are calibrated internally by minimizing the distance between 16 empirical moments and their equilibrium counterparts in the model. Table 2 lists the targeted moments, their empirical values, and their simulated values from the model. Even though every targeted moment is determined simultaneously by all the parameters, in what follows we discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power.

We choose the size of the labor force \( \bar{L} \) to match an average firm size of 17.8 employees, as computed from the Business Dynamics Statistics (BDS) over the period 2001-2007. We set the
flow of home production of the unemployed $\pi$ to replicate a steady-state elasticity of measured market tightness $\theta$ to a permanent shift in aggregate productivity of 23. This way, the model generates a realistic amount of volatility of labor market variables in response to real shocks.\\footnote{Remember that one our objectives is to explore the differential response of the model to productivity and financial shocks and, to do so, it seems reasonable to start from an economy where productivity shocks induce empirically plausible aggregate fluctuations. Put differently, accounting for the observed large movements of unemployment and vacancies along the Beveridge curve is a necessary starting point for assessing the ability of the model to produce outward movements of the Beveridge curve.}\textsuperscript{25} We choose the shift parameter of the matching function (a normalization of the value of $\Phi$ in steady state) and the exogenous quit rate $\zeta$ in order to pin down worker flows (net of flows involving temporary layoffs), i.e., a monthly job finding rate of 0.265 and a monthly separation rate of 0.02. Together, these two moments yield a steady-state unemployment rate of 0.07.

Together with $\zeta$ the parameters $\delta$, $\chi_0$ and $\chi$ also have a large effect on worker and job flows.
Note that δ is a way of destroying large and old firms and so impacts survival rates at old ages, hence we target the five-year survival rate of 50 percent found in BDS data. The amount of endogenous firm destruction is governed by the operation cost χ which moves exit rates of young firms, therefore we target the annual entry-rate (in steady-state the same as the annual exit-rate) of 11 percent. Finally the behaviour of entering firms is determined by the productivity of entrants, hence we choose a value for the set-up cost χ₀ to match the annual BDS fraction of job-creation by entrants of 18 percent.²⁶

We assume the revenue function \( y(z, n) = zn^\nu \) and introduce a small degree of permanent heterogeneity in the parameter \( \nu \). Specifically we consider a three-point distribution with support \{\( \nu_l, \nu_m, \nu_h \)\} that is symmetric about \( \nu_m \). We therefore have three parameters to choose: (i) the value of the \( \nu_m \), (ii) the distance \( \nu_h - \nu_l \) and (iii) the weight on \( \nu_m \). This allows the model to match the firm size distribution in the same spirit as the use of permanent heterogeneity in productivity in the quantitative applications of Elsby and Michaels (2013) and Kaas and Kircher (2011). As noted by Sedláček (2014) the assumption of heterogeneity in \( \nu \) is useful in a model of entry and exit since it can generate old, small firms while differences in permanent productivity would induce a counterfactually high correlation between firm age and size. This also allows us to quantitatively replicate the findings in Haltiwanger, Jarmin, and Miranda (2012) that age rather than size determine growth rates. Values of these three parameters are chosen to match a profit share of output of 5 percent and statistics from the BDS showing that firms with more than 500 workers account for 0.006 percent of firms but 47 percent of employment.

We postulate an AR(1) process for the firm-level productivity shock z. We calibrate the standard deviation and the persistence of the shocks to match the standard deviation of employment growth for continuing establishments in the cross section, 0.42 at the annual frequency, and the fraction 0.41 of continuing firms with an annual growth rate of less than 0.05.²⁷ We set the initial productivity distribution for entrants \( \Gamma_0 \) to the stationary distribution implied by the AR(1) process for incumbent productivities.²⁸

²⁶When computing moments designed to be comparable to their counterparts in the BDS we carefully time-aggregate the model to an annual frequency. For example, the entry-rate in the BDS is measured by the number of age zero firms in a given year divided by the total number of firms. Computing this statistic in the model requires allowing for monthly entry and exit for 12 months.

²⁷Both statistics are taken from the LBD by Elsby and Michaels (2013).

²⁸This seems both quantitatively and empirically reasonable. Quantitatively, if we were to choose a Pareto or similar distribution for \( \Gamma_0 \) then although initial productivities would lead to high rates of early growth, average
We now turn to hiring costs. The cost function (18) has four parameters: the two cost shifters \((\kappa_1, \kappa_2)\) and the two elasticities \((\gamma_1, \gamma_2)\). We choose these to match four moments. First, a cross-sectional elasticity of job filling rates to employment growth rates of 0.82, as DFH estimate.\(^{29}\) Second, the ratio of the average size of young \((\leq 5\) years) and old \((> 5\) years) firms of 1.96 from the BDS. Third, the ratio of recruiting intensity of small firms \((< 50\) employees) to large firms \((> 50\) employees), equal to 1.5 from JOLTS data on hires and vacancies by firm size. Fourth, the total hiring cost as a fraction of monthly wage per hire, according to Silva and Toledo (2009), is 0.4. We motivate our choice of moments as follows: the first moment almost exactly pins down the ratio of \(\gamma_1/\gamma_2\), the relative curvature of average vacancy-costs in terms of effort and the vacancy-rate, the second moment then pins down the overall curvature of the cost-function by matching the average rate of firm growth. Similarly the third moment is informative as to the ratio of \(\kappa_1/\kappa_2\), while the fourth shifts overall hiring costs relative to the wage.

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\(^{29}\)We cannot map directly \(\gamma_2/(\gamma_1 + \gamma_2)\) into the value estimated by DFH since in DFH the growth rate is the Davis-Haltiwanger growth rate normalized in \([-2, 2]\).
Figure 6: Life-cycle averages of various firm-level variables. Panels A, B, C, D, F computed by age for all incumbent firms. Panel E computed conditional on hiring.

4.1 Cross-sectional implications

We now explore some cross-sectional implications of the calibrated model, at its steady-state equilibrium.

In Figure 6 we plot the average firm size, growth rate, job creation and destruction rates, recruiting intensity, and debt for firms from birth through to maturity. Panel A shows that the convex recruiting cost and the dividend constraint slow down growth: the optimal firm size is reached, on average, after 7-8 years or so. Growth rates are highest for young firms and this is reflected in the life-cycle path of job creation rates (panels B and C). The destruction rate is rather flat over the life-cycle, but exit accounts for a larger share of destruction for young firms. Thus, our model produces a life cycle that is qualitatively consistent with the “up or out”
dynamics of young firms documented in the literature (Haltiwanger, 2011).

Recruiting intensity is sharply decreasing with age. This is what we should expect since our cost function implies that recruitment effort is increasing in the growth rate and young firms are those with the highest desired growth rates. Since the recruiting cost is highly convex in the vacancy rate, young firms find it optimal to limit the number of new positions but recruit very aggressively for the ones they choose to open. Panel D illustrates that debt is an important tool for young firms with high growth prospects. This small fraction of firms are constrained by the limited availability of equity and borrow aggressively to finance growth. After this initial phase, they start repaying and debt falls.

Figure 7 shows how recruitment and vacancy decisions interact with the distribution of firms in steady-state. Relative to the steady-state distribution of firms over age, the effort distribution is skewed towards young firms while the vacancy distribution is skewed towards older firms. These cross-sectional relationships generated by the model in steady-state provide the foundation for the mechanism we proposed in the introduction. Namely that recruiting intensity is chiefly determined by the behaviour of young, fast growing firms and that the birth and growth of these firms are susceptible to worsening in financial conditions due to their dependency on debt to finance growth.
5 Experiments (preliminary)

The aim of our numerical experiments is to account for the dynamics of some key labor market variables during the 2007-09 recession and throughout the slump that followed. In order to put our findings in context, and to better isolate the role of financial shocks—germane to the last recession—we contrast the predictions of the model for the Great Recession to those for the 2001 downturn. Figure 8 plots the evolution of vacancies, unemployment, job finding rate, vacancy yield, and average recruiting intensity $\Phi$—our index of aggregate matching efficiency—around these two contractionary episodes.

We interpret the data as follows. First note that, a year into either recession, unemployment had increased by 40 percent and vacancies had fallen by around 40 percent. Yet in 2007-09 the drop in match efficiency was much more severe. Given similar degrees of labor market slack, the larger fall in aggregate match efficiency generates a shallower profile for vacancy yields and a sharper decline in the job-finding rate. Moving forward, in the last recession match efficiency remained persistently low and, as a consequence, the vacancy-yield resembles that of 2001—despite increasingly more slack in the labor market—job-finding rates continue to fall, and unemployment continues to increase.
Our experiments trace the dynamics of the model economy starting from its initial steady state—described in Section 4—and transiting deterministically to a new steady state. We are interested in the aggregate response of the series in Figure 8 to combinations of three surprises—a real shock to aggregate TFP, and two financial shocks: an increase in the fraction $1 - \varepsilon$ of the set-up cost that start-ups must borrow to begin operating, and a rise in the financial wedge $\varphi$.

The fall in $\varepsilon$ is meant to summarize a dimension of financial shocks especially pertinent to the last recession. As argued in the Introduction, startups and young firms do not have an established credit record and, as such, they often rely on personal sources of finance, including home equity. Thus, a fall in housing prices is likely to worsen the entrepreneurial need for external funding in order to start a new business and increase financing costs.

The increase in $\varphi$ is meant to capture a disruption of the financial sector that escalates credit spreads, over and above the increase in default risk. Gilchrist and Zakrajšek (2012) estimate credit spreads for the U.S. corporate sector over the past 40 years and separate the component associated to time-varying default risk from the excess bond premium, a component that represents variation in the pricing of default risk, rather than in the risk of default itself, and thus provides a useful gauge of aggregate credit supply conditions. As argued in the parameterization section, in our model, movements in $\varphi$ map into changes in the measured excess premium.

In light of this discussion, in our first experiment, we simulate a recession that mimics the 2001 downturn. We think of this recession as entirely driven by a decline in TFP of about 3 percentage points over 1.5 years, and a gradual recovery back to its pre-recession level after 6 years. In our second experiment, we engineer the same decline in TFP and, on top of it, we reduce $\varepsilon$ enough to replicate a fall of 27 percent in the number of start-up firms. We then slowly increase $\varepsilon$ back to its initial level in a way that reproduces the dynamics of the entry rate in the post-recession years. In our third experiment, besides the same fall in TFP, we increase $\varphi$ by 400 basis points in the year after the onset of the recession, and we slowly bring them back to their initial level over the next two years. This path replicates the evolution of the excess bond premium.

\footnote{\textit{Gilchrist and Zakrajšek} (2012) report, for example, a strikingly high correlation between their measure of excess bond premium and diffusion index of the change in credit standards on commercial and industrial loans at U.S. commercial banks obtained from the Federal Reserve’s quarterly Senior Loan Officer Opinion Survey on Bank Lending Practices.}

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premium estimated by Gilchrist and Zakrajšek (2012) over the period 2007-2010.\textsuperscript{31}

### 5.1 2001 recession

The model produces similar paths for unemployment and the job-finding rate as observed in the 2001 recession. However, these paths are generated by a smaller decline in vacancies and a smaller increase in the vacancy-yield. This is due to an excessive response of aggregate recruiting intensity to the productivity shock which we discuss in section 5.2.

### 5.2 2007-09 recession

The economy responds strongly to the productivity shock augmented with a decrease in $\varepsilon$. Despite being a shock only to potential entrants and impacting incumbent firms only through movements in the equilibrium value of $\theta^*$, the fall in $\varepsilon$ generates an additional 40 percent increase in unemployment and an additional 40 percent decline in the job finding rate. This result is due to the large fraction of job-creation from young firms and their high exposure to financial

\textsuperscript{31}In the current version of the paper, we only consider the first and second experiments described. For now, we also choose a decrease in $\varepsilon$ to 0.10 as opposed to matching the exact path of entrants. For the series of $\varepsilon_t$ and $Z_t$ used in the experiments shown here see Appendix B.
shocks. The large drop in entry generated by the shock means that the model can reproduce the persistence of the crisis, with unemployment still 20 percent above its pre-recession level after six years, despite the values of the shocks having returned to their steady-state levels — entry rates are lowest around 12 months generating a large missing-cohort effect 5 years later when these firms would typically account for around 30 percent of employment.

Finally, the model matches the decline in aggregate match efficiency found in the data. This restrains the increase in vacancy yields and contributes to a large rise in unemployment relative to the more modest decline in vacancies. In other words, if mapped into $UV$-space the model generates a large outward shift in the Beveridge curve.

Our model-data comparisons of the recessions generated by our current configuration of shocks have shown that the model can match the drop in aggregate matching efficiency observed in 2008 at the cost of generating a counter-factually large decrease in match efficiency in 2001. Figure 11 sheds light on why this is so. In panel A we see that the model implies a 10 percent decline in entry in 2001 whereas in the data the number of new firms increases. A more accurate experiment would be that which we initially proposed: a configuration of the paths for $\varepsilon$ to match the realised path for entry. In the case of 2001, we would require a small increase in $\varepsilon$, giving the interpretation of the recession as a TFP shock during a period of financial liberalisa-
tion. This interpretation seems empirically plausible given that house prices and the ability to use housing to fund equity capital in start-ups was increasing throughout the 1990s and 2000s. Such a correction would contain the decline in aggregate match efficiency. Regarding the 2008 recession, our decline in $\varepsilon$ appears excessive with respect to the entry-rate in terms of the level yet conservative with respect to its persistence. In further experiments, we will choose the path of $\varepsilon$ to match this decline.

6 Conclusions

In this paper we have proposed a novel mechanism to explain outward shifts in the Beveridge curve (i.e., falls in aggregate match efficiency) based on fluctuations in firms’ aggregate recruiting intensity. The model formalizes an intuitive link between financial frictions in the availability of start-up equity and in loan markets, the hiring behaviour of young firms and their effort in recruitment. A deterioration of financial conditions has disproportionate effects on start-ups and young firms which are those with the highest recruiting intensity. When aggregated into a plausibly calibrated equilibrium model, this firm-level behaviour leads to sizable movements in aggregate matching efficiency in response to financial shocks. Crucially, the calibrated model’s cross-section is consistent with the empirical relation between job filling rates and firm-level growth rates in micro data.
References


Appendix

This Appendix is organized as follows. Section A contains the derivations of the cost hiring function that we introduced in Section 4. Section B contains one figure showing the paths of the exogenous variables used in the computation of transitional dynamics in Section 5.

A The hiring cost function

In this section we show that, once we postulate the hiring cost function

\[ C(n, e, v) = \left[ \frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left( \frac{v}{n} \right)^{\gamma_2} \right] v \]  

(A1)

then, through firm’s optimization we obtain a log-linear cross-sectional relationship between the job-filling rate and the employment growth rate that is consistent with the empirical findings in DFH. Next, we show that our cost function boils down to the one that Kaas and Kircher (2011) choose. Finally, by substituting the firm FOCs into (A1), we derive a formulation of the cost only in terms of \((n, n')\) that we use in the intertemporal problem (17) in the main text.

As we explained in Section (3.1), the firm solves a static cost minimization problem: given a choice of \(n'\), it determines the lowest cost combination of \((e, v)\) that can deliver \(n'\). The hiring firm’s cost minimization problem is

\[ C(n, n') = \min_{e,v} \left[ \frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left( \frac{v}{n} \right)^{\gamma_2} \right] v \]  

(s.t. \( n' - n \leq q (\theta^*) e v \) \quad \text{e} \in [0,1], \quad v \geq 0) \]  

(A2)

Convexity of the cost function (A1) in \((e, v)\) requires \(\gamma_1 \geq 1\) and \(\gamma_2 \geq 0\). When \(\gamma_1 = \gamma_2 = 0\), we have the standard model where every firm sets \(e = 1\) and the cost of vacancy creation is linear. After setting up the Lagrangian, and ignoring for now the corner solution \(e = 1\), one can easily derive the two FOCs with respect to \(e\) and \(v\) that, combined together, yield a relationship
between the optimal choice of \( e \) and the optimal choice of the vacancy rate \( v/n \):

\[
e = \left[ \frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1}} \left( \frac{v}{n} \right)^{\frac{\gamma_2}{\gamma_1}}. \tag{A3}
\]

Note that, if \( \gamma_2 = 0 \), as in Pissarides (2000), recruiting intensity is equal to a constant for all firms and it is independent of aggregate labor market conditions—both counterfactual implications. The following changes in parameters (ceteris paribus) result in a substitution away from vacancies and towards effort: \( \uparrow \kappa_2, \downarrow \kappa_1, \uparrow \gamma_2, \) and \( \downarrow \gamma_1 \). The effect of the cost shifter is obvious. A higher curvature on the vacancy rate in the cost function (\( \uparrow \gamma_2 \)) makes the marginal cost of creating vacancies rising faster than the marginal cost of recruiting effort; since the gain in terms of additional hires from a marginal unit of effort or vacancies is unaffected by \( \gamma_2 \), it is optimal for the firm to use relatively more effort.

Now, substituting the law of motion for employment at the firm level into (A3), we obtain the optimal recruitment effort choice, expressed only as a function of the firm-level variables \((n, n')\):

\[
e(n, n') = \left[ \frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_2}{\gamma_1 + \gamma_2}} \left( \frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}. \tag{A4}
\]

which, in turn implies, for the job filling rate,

\[
f(n, n') = q(\theta^*) e(n, n') = \left[ \frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left( \frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}. \tag{A5}
\]

This equation demonstrates that the model implies a log-linear relation between the job filling rate and employment growth at the firm level, with elasticity \( \gamma_2 / (\gamma_1 + \gamma_2) < 1 \) as in the data. Moreover, firm-level job filling rates are countercyclical, through their dependence on \( q(\cdot) \).

Finally, using (A5) into the firm-level law of motion for employment yields an expression for the vacancy rate

\[
\frac{v}{n} = \left[ \frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left( \frac{n' - n}{n} \right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}}. \tag{A6}
\]

Now, note that, by substituting the optimal choice for recruitment effort (A3) into (A1), we
obtain the following formulation for the cost function

\[ C(n, v) = \left[ \kappa_2 \frac{\gamma_1 + \gamma_2}{(\gamma_1 - 1)(\gamma_2 + 1)} \left( \frac{v}{n} \right)^{\gamma_2} \right] v, \quad (A7) \]

which is one of the specifications that Kaas and Kircher (2011) invoke.

Finally, if we use \((A6)\) into \((A7)\), we obtain a version of the cost function only as a function of \((n, n')\) that we can use directly in the dynamic problem \((17)\):

\[ C^*(n, n') = \kappa_2 \left[ \frac{\gamma_1 + \gamma_2}{(\gamma_1 - 1)(\gamma_2 + 1)} \right] \left\{ \frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \frac{1}{\gamma_1 + \gamma_2} q(\theta^*) - \frac{\gamma_1}{\gamma_1 + \gamma_2} \left( \frac{n' - n}{n} \right) \frac{\gamma_1}{\gamma_1 + \gamma_2} \right\}^{1+\gamma_2} n. \]
B Transition dynamics shocks

Figure B1: Transition dynamics — Shocks used in computations